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Cosmographic tests of the Hubble law

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GR18, Sydney
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The original Hubble law:

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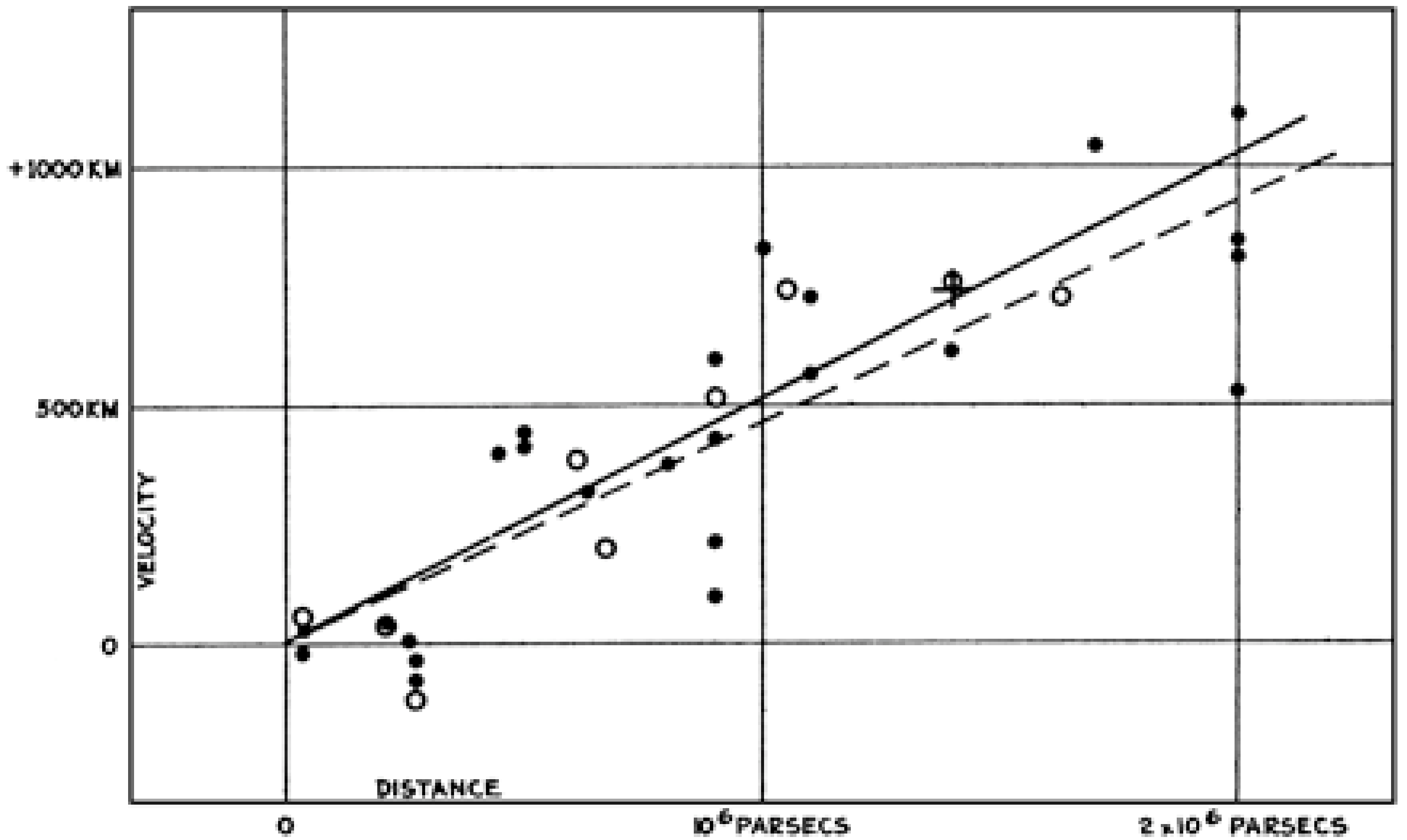
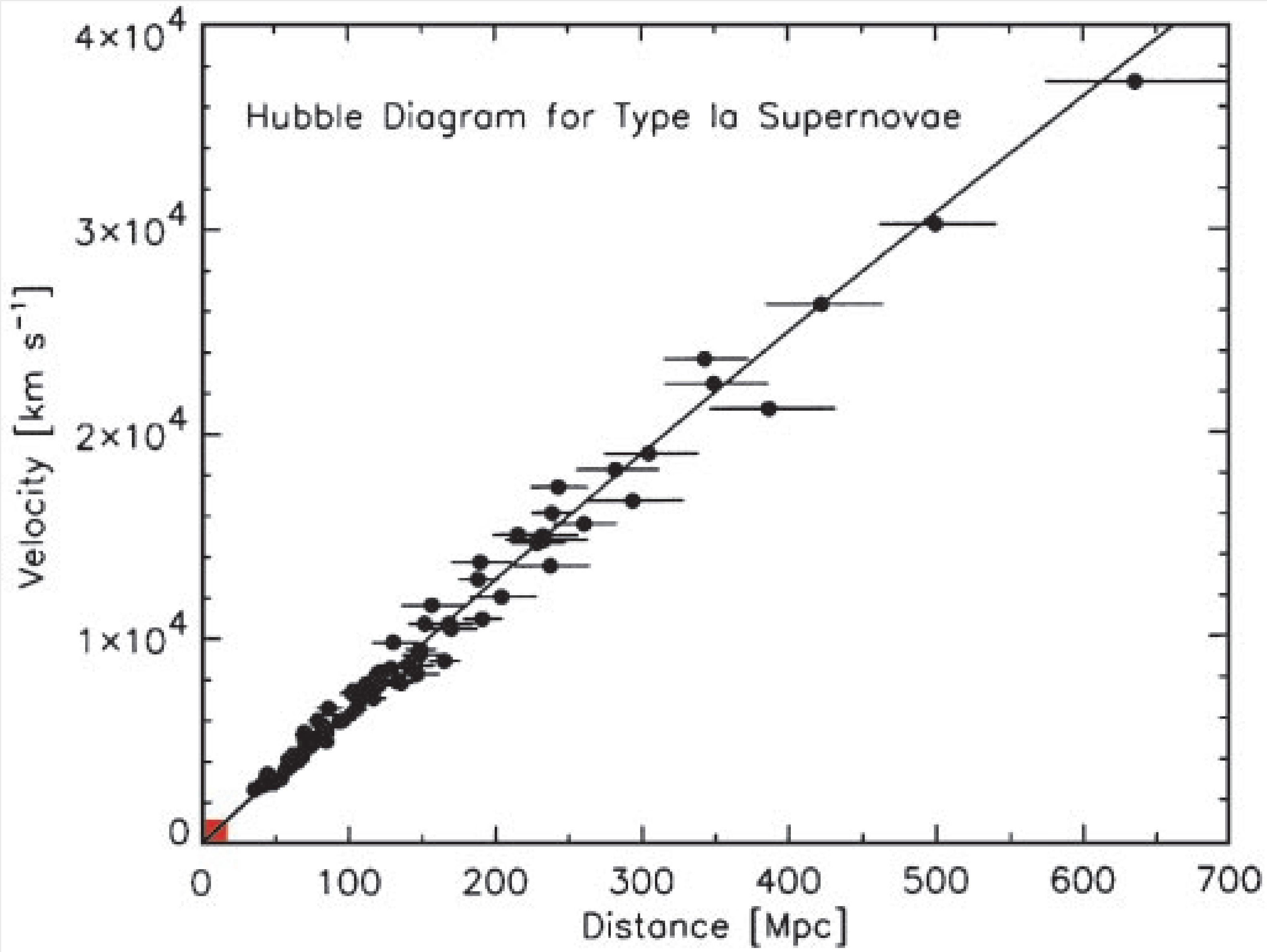
$$v = H_0\, d; \qquad H_0 \approx 500\ (\text{km/sec})/\text{Mpc}.$$



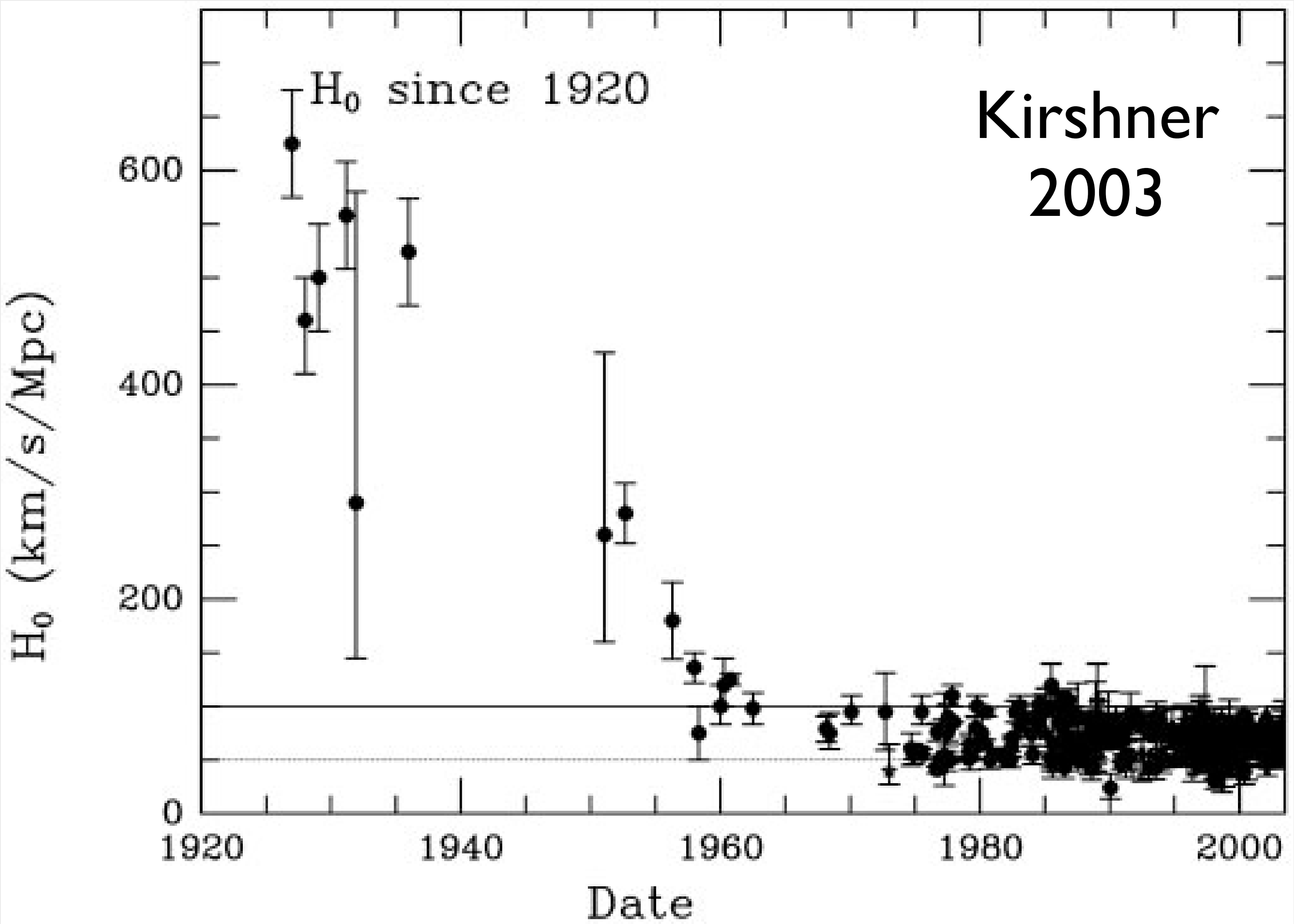
1929

[Hubble, E. P. (1929) *Proc. Natl. Acad. Sci. USA* 15, 168–173]

2003



The Hubble diagram for type Ia supernovae. The scatter about the line corresponds to statistical distance errors of < 10% per object. The small red region in the lower left marks the span of Hubble's original Hubble diagram from 1929. [Kirshner 2003]



Published values of the Hubble parameter versus time.
At each epoch, the estimated error in the Hubble parameter is small compared with the subsequent changes in its value.
This is a symptom of underestimated systematic errors.

Speed of light since 1880:

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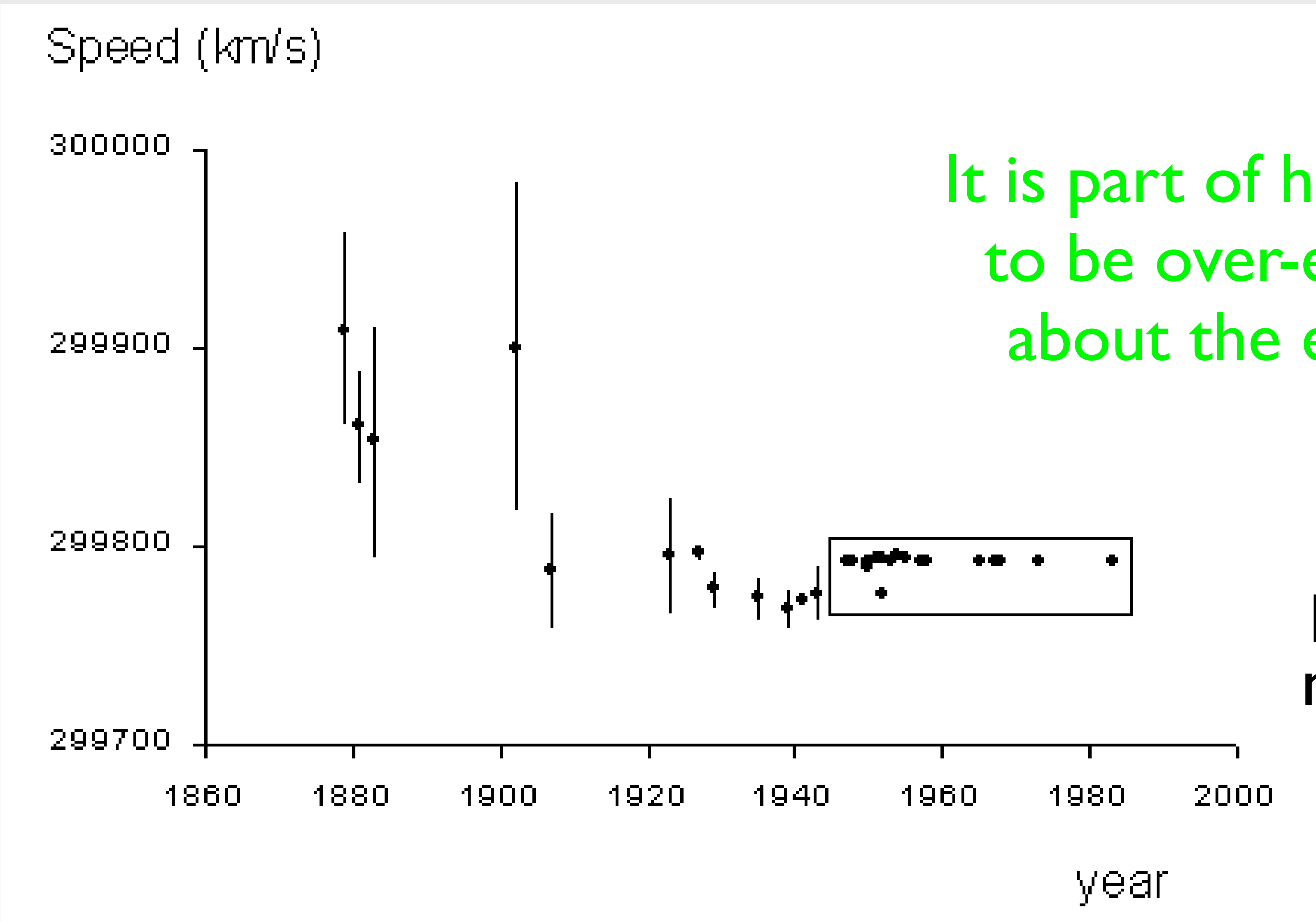

It is part of human nature to be over-enthusiastic about the error bars.

This phenomenon not limited to cosmology...

Speed of light since 1940:

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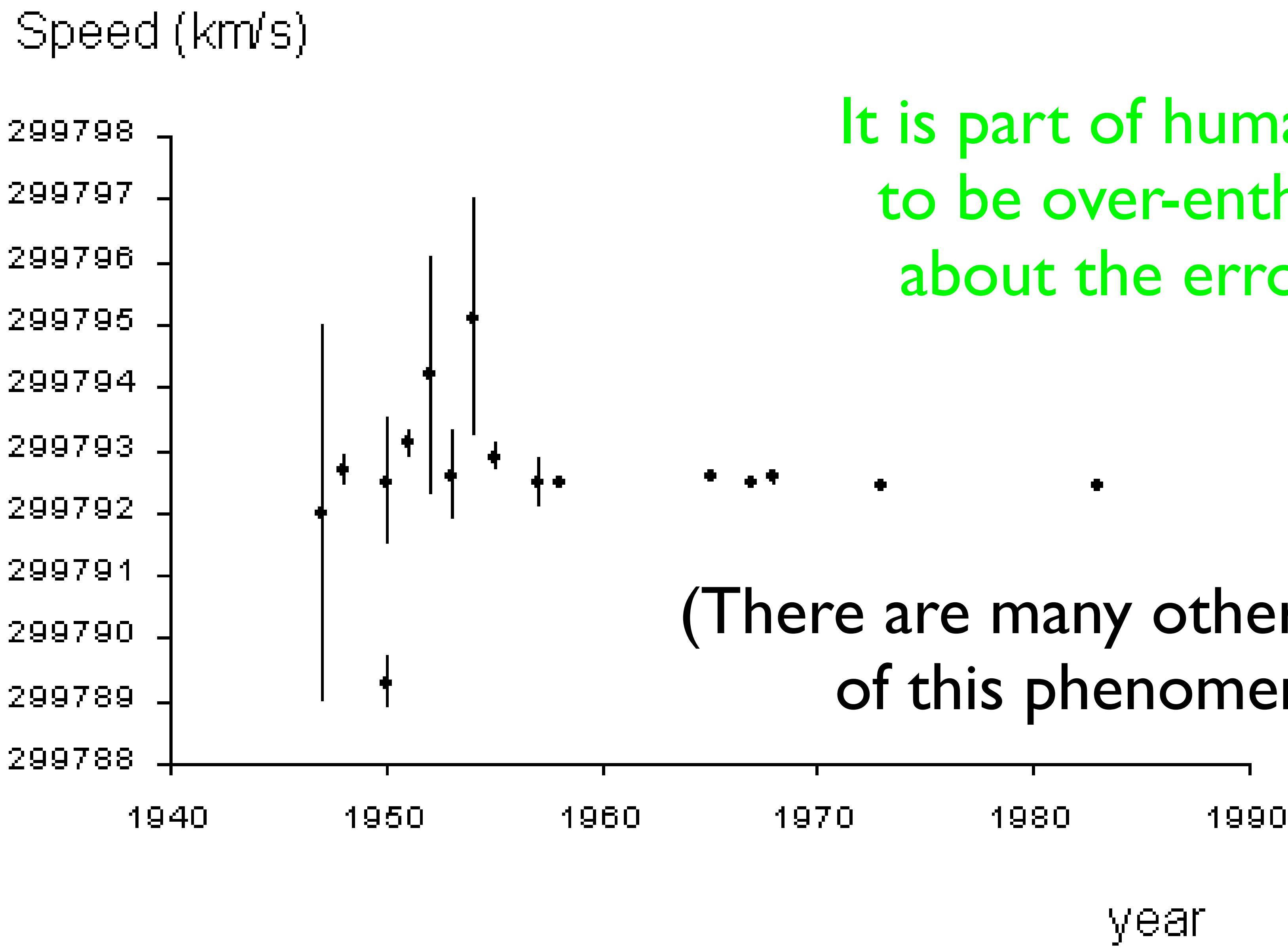
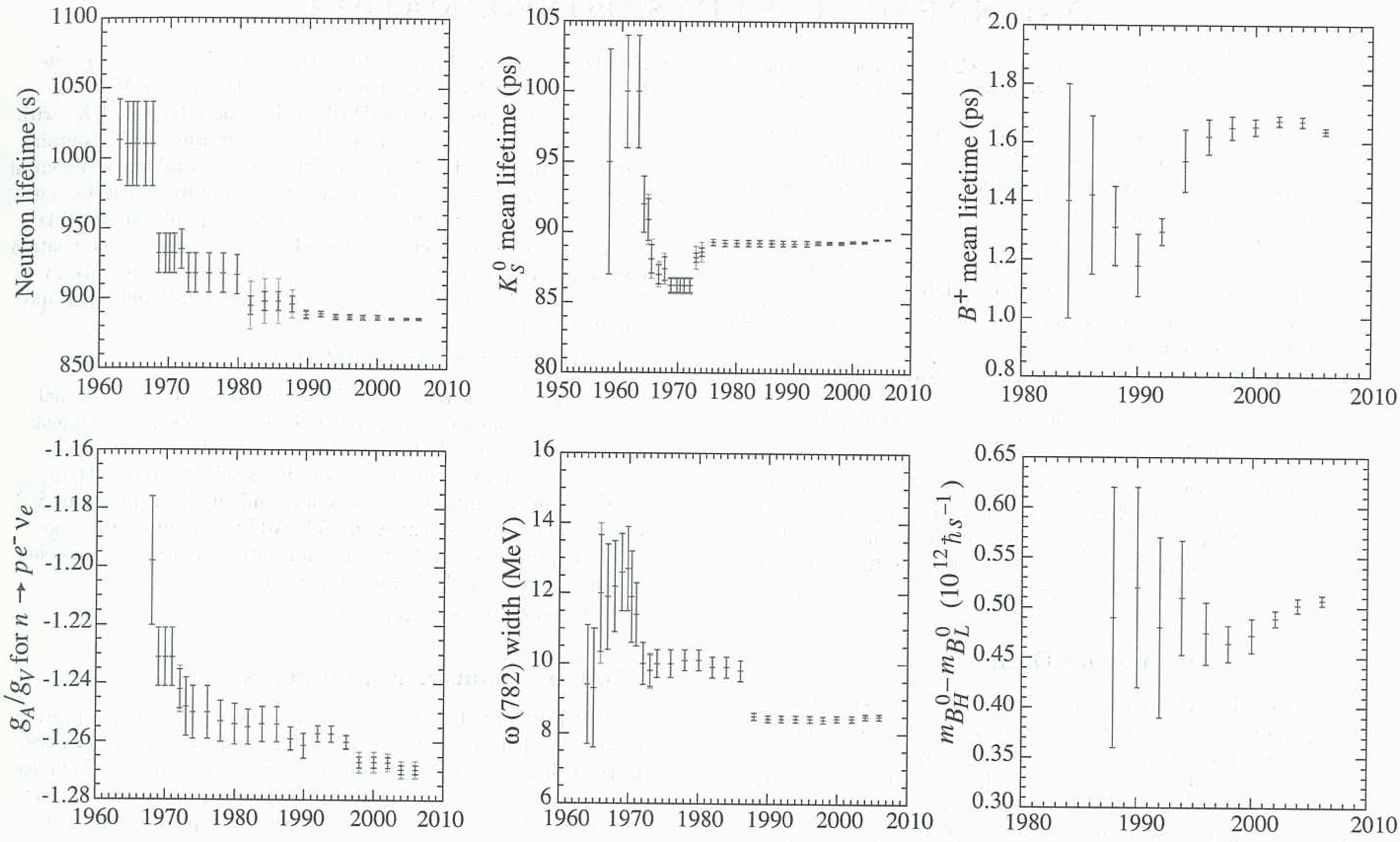


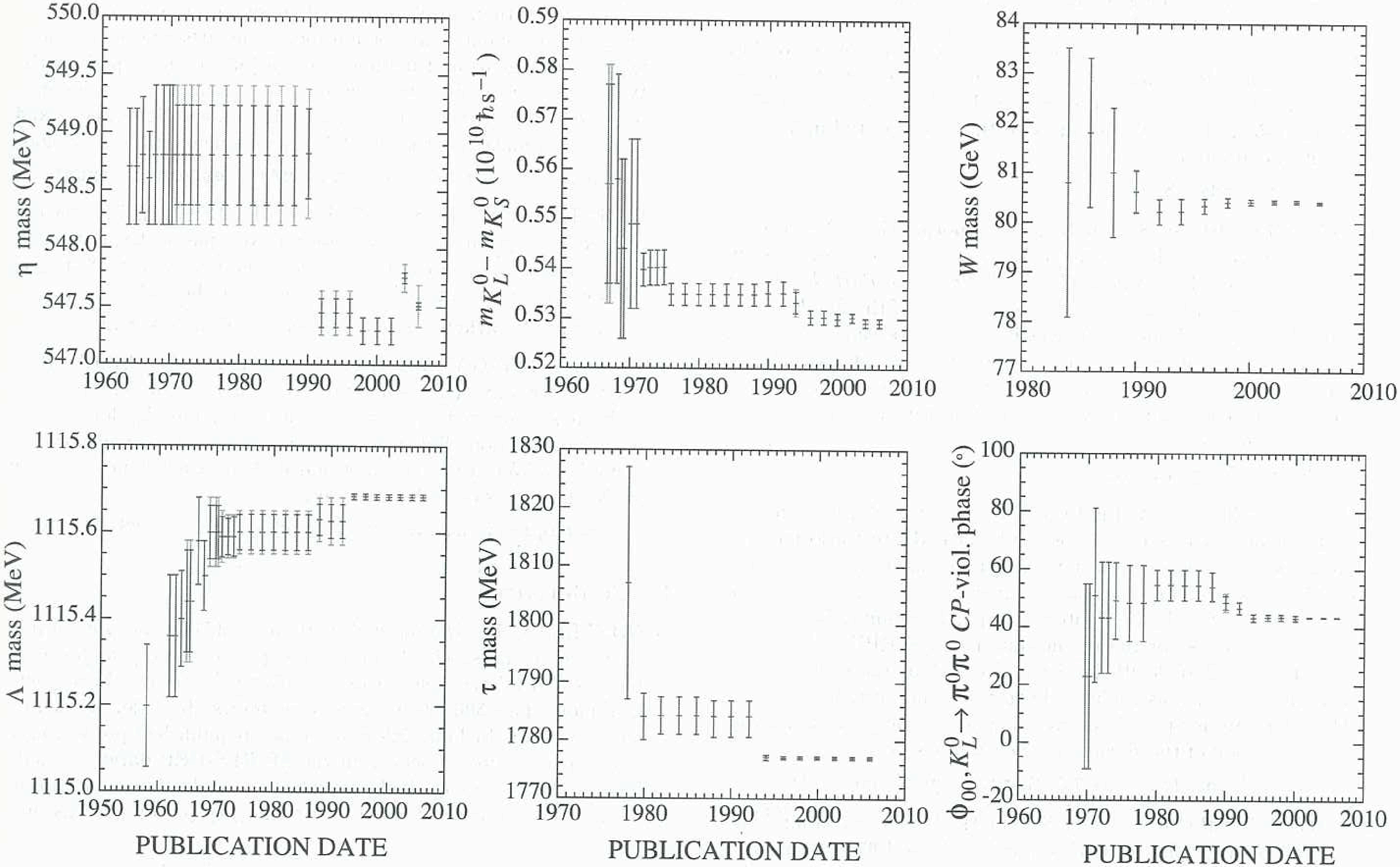
It is part of human nature to be over-enthusiastic about the error bars.

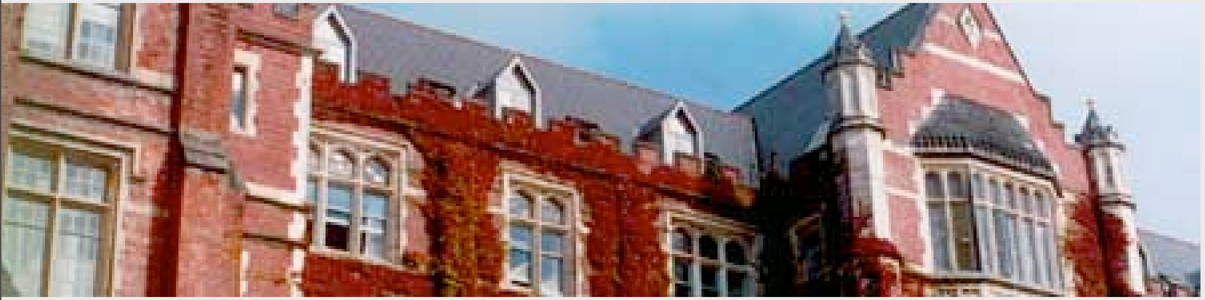
(There are many other examples of this phenomenon.)

Selected measurements from particle physics:



Selected measurements from particle physics:





Modern version of Hubble law:

[for example: Chiba, Sahni, Visser]

$$d_L(z) = \frac{c\,z}{H_0}\left\{1 + \frac{1}{2}\left[1 - q_0\right] z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2\,a_0^2}\right] z^2 + O(z^3)\right\}.$$

"Jerk" parameter:

$$j(t) = \frac{\dddot{a}(t)\;a(t)^2}{\dot{a}(t)^3}; \qquad j_0 = \frac{\dddot{a}(t_0)\;a(t_0)^2}{\dot{a}(t_0)^3}.$$

Higher-order expansions are possible... [Visser]

Modern tests:

Latest tests of the Hubble law are based largely on supernova data, approximately 200 supernovae.

Now have data out to redshift: $z \sim 1.75$

Major datasets:
- Gold+Silver+Nearby (gold06)
- Supernova Legacy Survey (legacy05)

Lots of little "quirks" hiding in the processed data.

Photon flux version of the Hubble law:

$$d_F(z) = d_H\ z \left\{ 1 - \frac{1}{2} q_0 z + \frac{1}{24}\left[3 + 10 q_0 + 12 q_0^2 - 4(j_0 + \Omega_0)\right] z^2 + O(z^3) \right\}.$$

[count photons, not energy]

[Visser, Cattoen]

Transform it:

$$\begin{aligned} \ln[d_F/(z\ \text{Mpc})] &= \frac{\ln 10}{5}[\mu_D - 25] - \ln z - \frac{1}{2}\ln(1+z) \\ &= \ln(d_H/\text{Mpc}) \\ &\quad - \frac{1}{2} q_0 z + \frac{1}{24}\left[3 + 10 q_0 + 9 q_0^2 - 4(j_0 + \Omega_0)\right] z^2 + O(z^3). \end{aligned}$$

--- simple probe for deceleration parameter

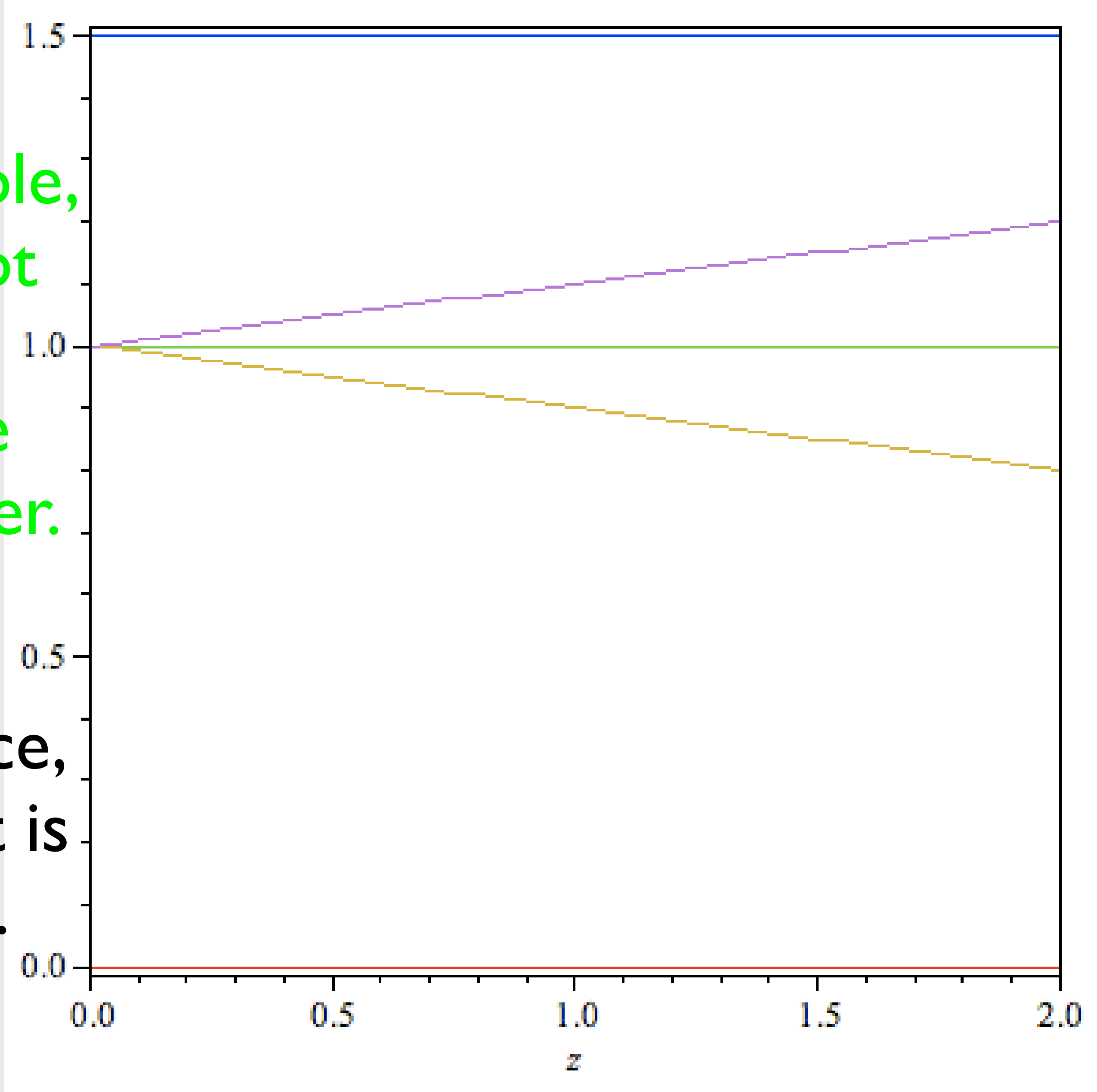
--- stellar magnitude and redshift provided in the data

--- plot the data...

We expect something like this:

In principle, intercept yields Hubble parameter.

In practice, intercept is noise...



Slope yields:

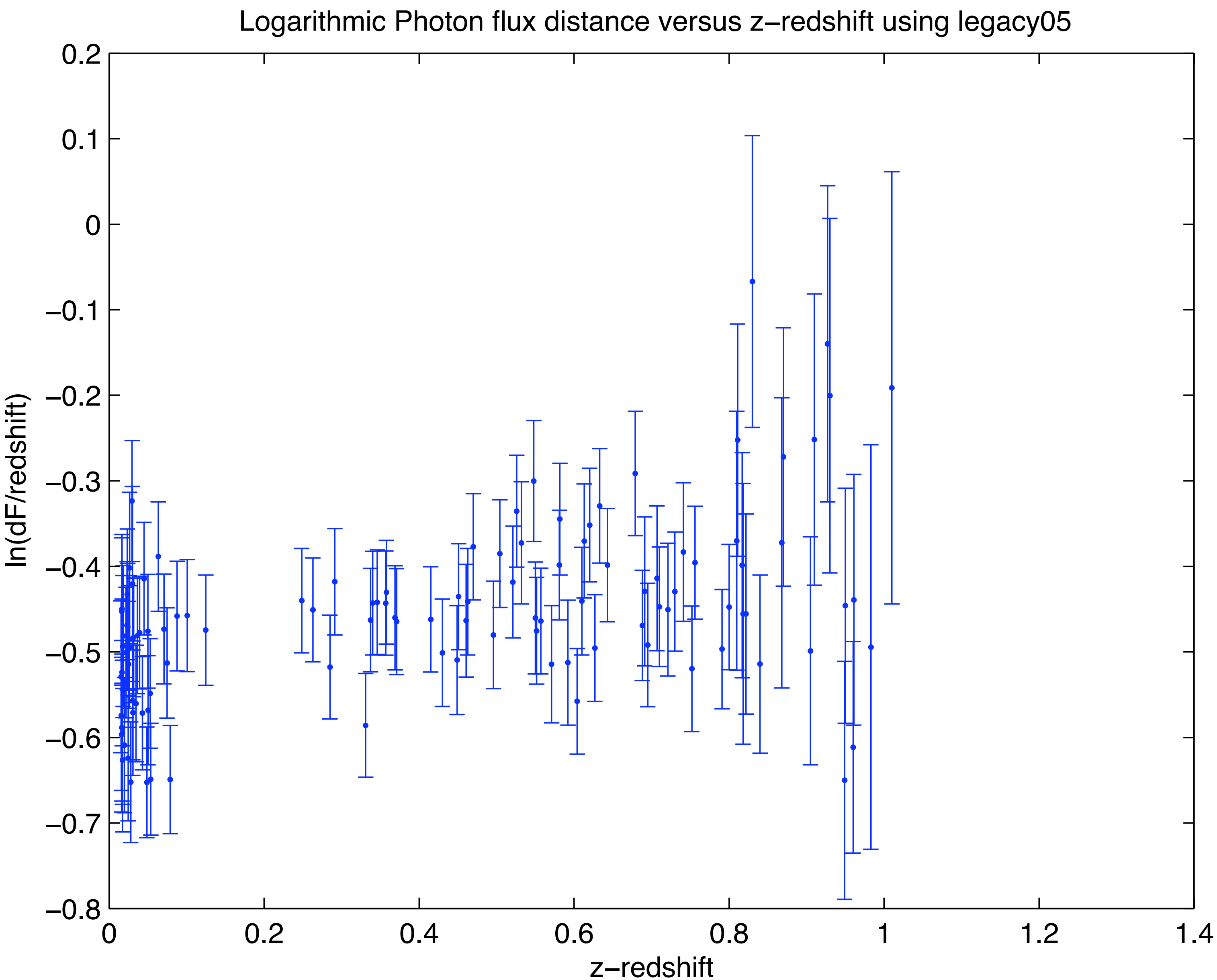
acceleration

coasting

deceleration

(overall calibration difficult...)

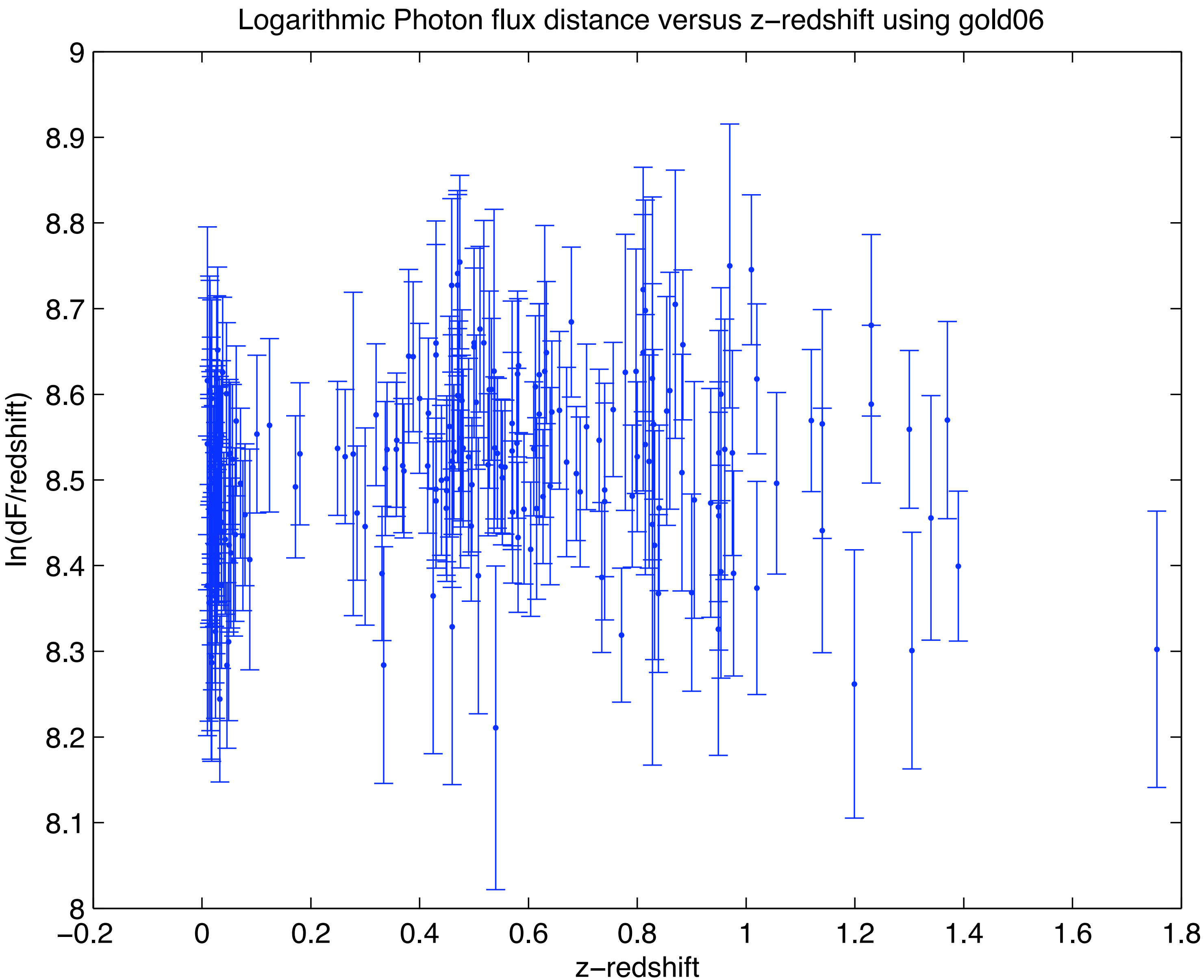
We get this:



legacy05

Smaller dataset, but homogeneous.

We get this:



gold06

Larger dataset, but not homogeneous.

Combined dataset from six different observing platforms.

Lies, damned lies, and statistics...

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The situation is actually worse than it looks because the plotted error bars report only part of the uncertainty...

The plots include photometric uncertainties plus "intrinsic variability" in the supernovae...

The plots do not include systematic uncertainties, neither "known unknowns" nor a budget for "unknown unknowns".

(This is traditional in cosmology...)

"Unknown unknowns" can be estimated historically...

Historical uncertainties:

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Most recent:

* As of 2006 the high redshift supernovae have all moved 5% closer than estimated in 2004.
  (Improved understanding and characterization of nonlinearities in the photodetectors.)

* Over the last decade there have still been 15% disagreements over the size of our own galaxy... (Hipparcos satellite data.)

* Hubble's mis-calibrated Cephid variables led to some 666% error...

NIST guidelines:

Type B evaluations of uncertainty:

"any method of evaluation of uncertainty by means other than the statistical analysis of a series of observations"

"A type B evaluation of standard uncertainty is usually based on scientific judgment using all of the relevant information available, which may include: previous measurement data, etc..."

NIST Technical Note 1297.

NIST recommended practice:

- Treat all uncertainties, whatever their source, "as though" they were statistical, and report an "equivalent one-sigma uncertainty"...

- Always combine uncertainties in quadrature, unless you have good reason to believe there is a correlation...

- In particular, combine statistical and systematic uncertainties in quadrature...

Modelling uncertainty:

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Ask the same statistical question several slightly different ways, and see if the estimates are close to each other...

The process of performing a least squares fit
does not commute
with the process of truncating a Taylor series...

((Trust me, you really do not want to see the relevant formulae))

[Cattoen, Visser, gr-qc/0703122]

legacy05 dataset

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distance	q_0	$j_0 + \Omega_0$
d_L	-0.48 ± 0.17	$+0.43 \pm 0.60$
d_F	-0.56 ± 0.17	$+1.16 \pm 0.65$
d_P	-0.62 ± 0.17	$+1.92 \pm 0.69$
d_Q	-0.69 ± 0.17	$+2.69 \pm 0.74$
d_A	-0.75 ± 0.17	$+3.49 \pm 0.79$

With 1-σ statistical uncertainties.

gold06 dataset

distance	q_0	$j_0 + \Omega_0$
d_L	-0.37 ± 0.11	$+0.26 \pm 0.20$
d_F	-0.48 ± 0.11	$+1.10 \pm 0.24$
d_P	-0.58 ± 0.11	$+1.98 \pm 0.29$
d_Q	-0.68 ± 0.11	$+2.92 \pm 0.37$
d_A	-0.79 ± 0.11	$+3.90 \pm 0.39$

With 1-σ statistical uncertainties.

Combine the analyses:

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dataset	redshift	$q_0 \pm \sigma_{\text{statistical}} \pm \sigma_{\text{modelling}}$
~~legacy05~~	~~y~~	~~$-0.66 \pm 0.38 \pm 0.13$~~
legacy05	z	$-0.62 \pm 0.17 \pm 0.10$
~~gold06~~	~~y~~	~~$-0.94 \pm 0.29 \pm 0.22$~~
gold06	z	$-0.58 \pm 0.11 \pm 0.15$

With 1-σ statistical uncertainties and 1-σ model building uncertainties, no budget for "systematic" uncertainties.

(We shall draw a veil of discrete silence over the unfortunate status of the jerk parameter.)

Include systematics:

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dataset	redshift	$q_0 \pm \sigma_{\text{statistical}} \pm \sigma_{\text{modelling}} \pm \sigma_{\text{systematic}} \pm \sigma_{\text{historical}}$
~~legacy05~~	~~y~~	~~$-0.66 \pm 0.38 \pm 0.13 \pm 0.09 \pm 0.09$~~
legacy05	z	$-0.62 \pm 0.17 \pm 0.10 \pm 0.09 \pm 0.09$
~~gold06~~	~~y~~	~~$-0.94 \pm 0.29 \pm 0.22 \pm 0.09 \pm 0.09$~~
gold06	z	$-0.58 \pm 0.11 \pm 0.15 \pm 0.09 \pm 0.09$

With 1-σ effective statistical uncertainties for all components.

I think you can see where this is headed...

(Some astrophysicists think we should provide even larger historical uncertainties.)

Combine uncertainties:

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$$\sigma_{\text{combined}} = \sqrt{\sigma^2_{\text{statistical}} + \sigma^2_{\text{modelling}} + \sigma^2_{\text{systematic}} + \sigma^2_{\text{historical}}}.$$

Expanded uncertainty: $U_k = k\,\sigma_{\text{combined}}$. [NIST]

Used when you need to be "certain" for either scientific or legal/ regulatory reasons...

Bitter experience in particle physics:

"If it's not 3-sigma, it's not physics..."

$U_3 = 3\,\sigma_{\text{combined}}$. [now 5-sigma?]

The 3-sigma standard:

Three-sigma corresponds to being 99.5% statistically sure you have a real effect...

Three-sigma is the minimum standard considered acceptable in particle physics before claiming "new physics"...

(This is of course a scientific judgment based on the historical record of what has worked in the past...)

The 3-sigma standard:

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dataset	redshift	$q_0 \pm \sigma_{\text{combined}}$	$q_0 \pm U_3$
~~legacy05~~	~~y~~	~~-0.66 ± 0.42~~	~~-0.66 ± 1.26~~
legacy05	z	-0.62 ± 0.23	-0.62 ± 0.70
~~gold06~~	~~y~~	~~-0.94 ± 0.39~~	~~-0.94 ± 1.16~~
gold06	z	-0.58 ± 0.23	-0.58 ± 0.68

That is: not statistically significant at three-sigma.

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Preponderance of evidence:

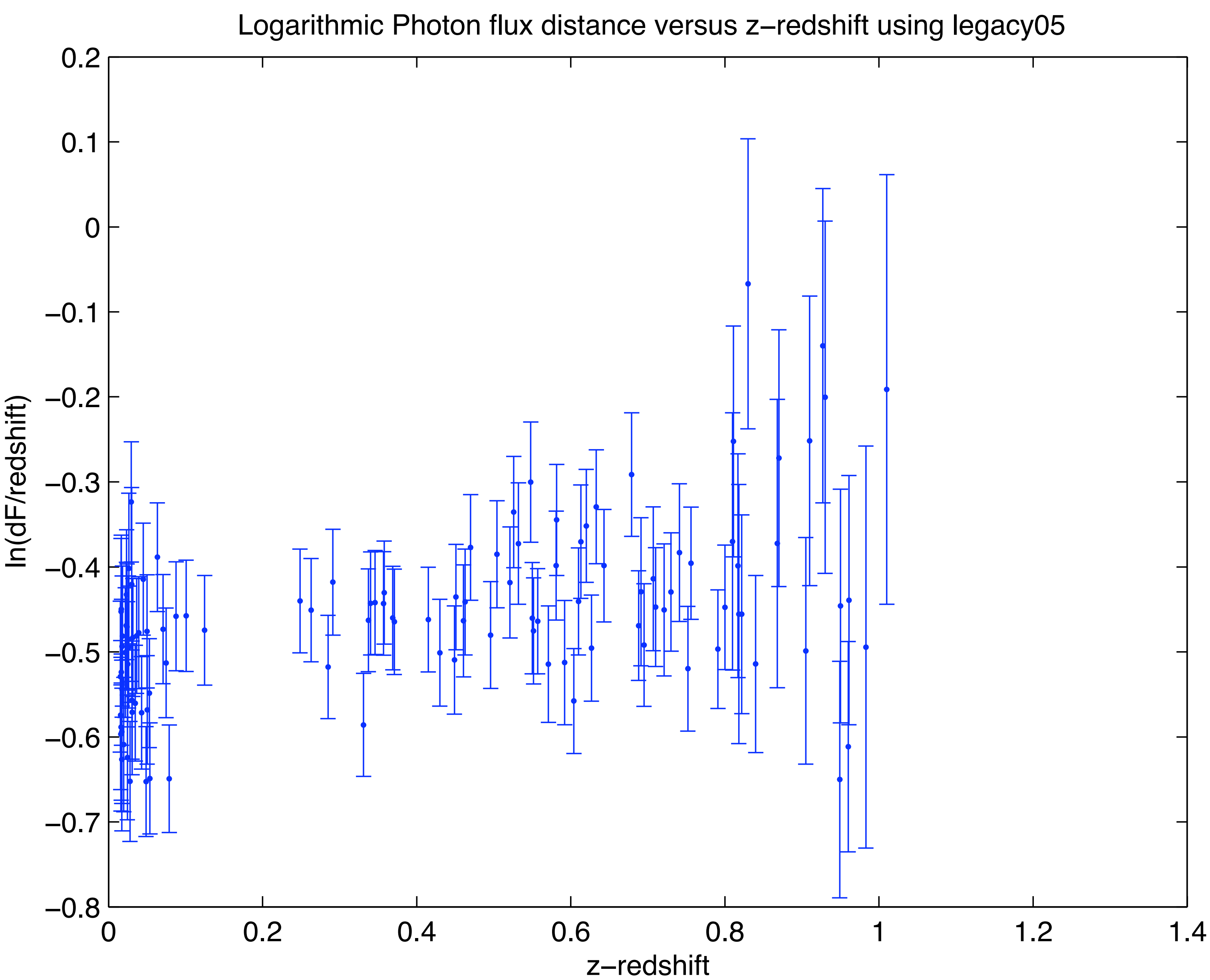
The universe is accelerating.

But (based on supernova data alone),
this acceleration is not established
“beyond reasonable doubt”.

There are an awful lot of subtleties hiding in the woodwork of the statistical analyses...

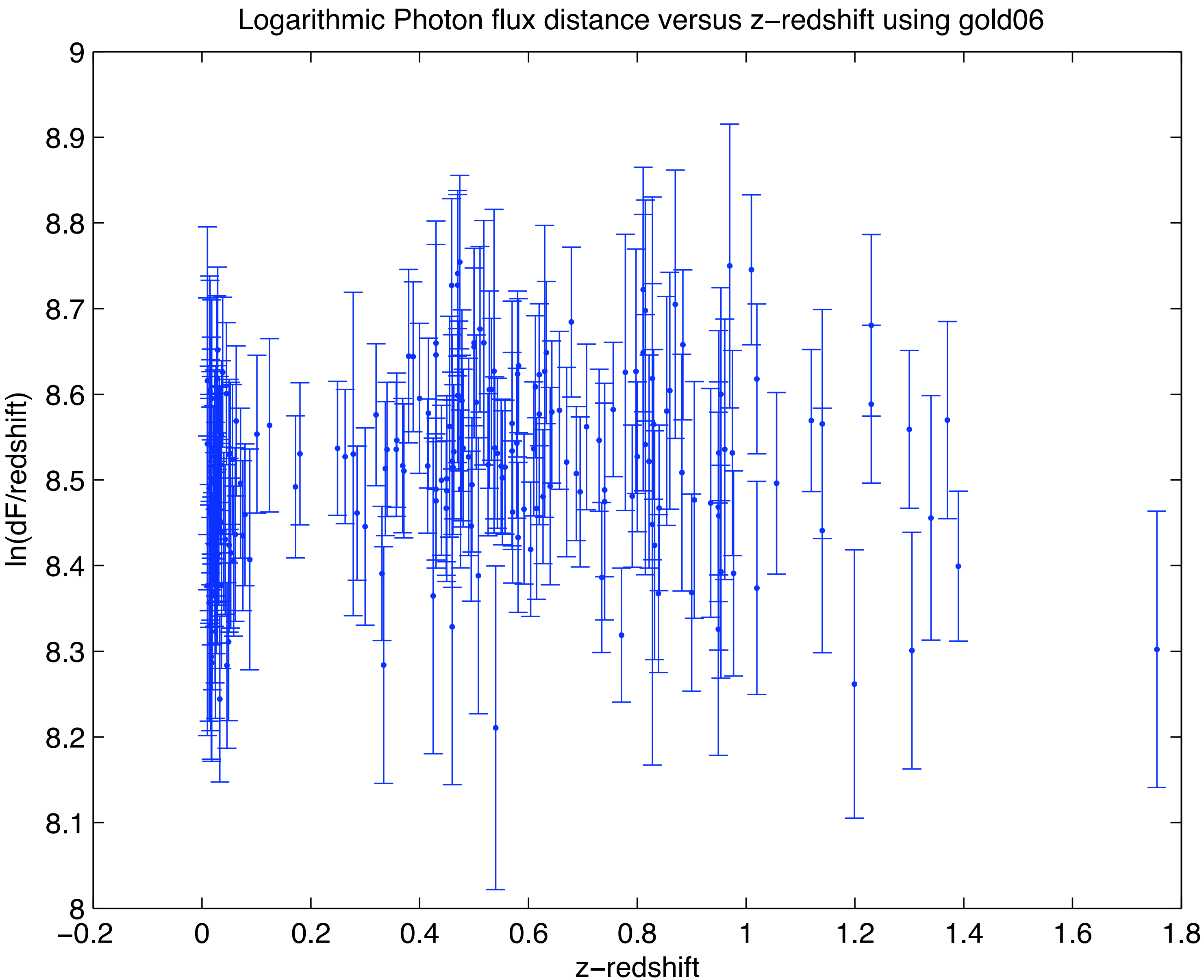
Antidote to excessive statistical sophistication:

Antidote:



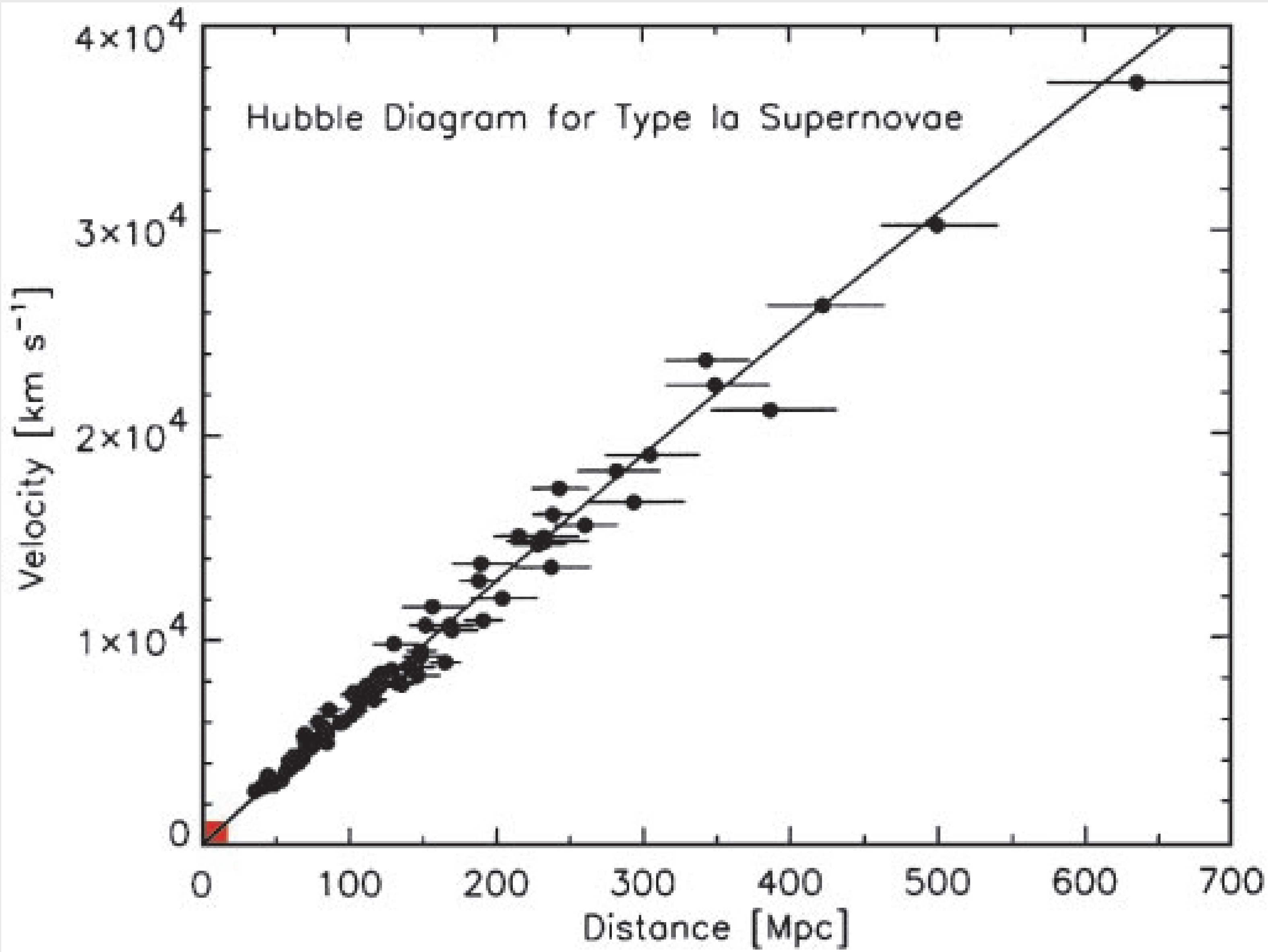
(statistical uncertainties only)

Antidote:



(statistical uncertainties only)

2003


Hubble Diagram for Type Ia Supernovae
Velocity [km s^{-1}]
4×10^4
3×10^4
2×10^4
1×10^4
0
0
100
200
300
400
500
600
700
Distance [Mpc]

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* Some parts of cosmology are already precision science.

* Cosmological distance determinations, however, are not yet precision science.

"Precision cosmology? Not just yet."

"It is important to keep an open mind; just not so open that your brains fall out"

--- Albert Einstein