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Minimal conditions for the creation of a Friedman–Robertson–Walker universe from a ‘‘bounce’’

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Abstract

In this Letter we investigate the *minimal* conditions under which the creation of our universe might arise due to a ‘‘bounce’’ from a previous collapse, rather than an explosion from a big-bang singularity. Such a bounce is sometimes referred to as a *Tolman wormhole*. We subject the bounce to a general model-independent analysis along the lines of that applied to the Morris–Thorne *traversable wormholes*, and show that there is always an open temporal region surrounding the bounce over which the strong energy condition (SEC) must be violated. On the other hand, *all* the other energy conditions can easily be satisfied. In particular, we exhibit an inflation-inspired model in which a big bounce is ‘‘natural’’.

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Oscillating universes [1,2] are alternatives to standard big bang cosmology [3–6]. They avoid the big-bang singularity and replace it with a cyclical evolution from a previous incarnation of our present universe. Unfortunately, many of the older discussions of oscillating universes leave the nature of the turnaround quite ambiguous (cusp? angular-momentum barrier?). Interest in oscillating universes largely declined after the development of the first cosmological singularity theorem [3,4], but we feel that the time is ripe for a reassessment of the situation. In this Letter, we model the turnaround by a Friedman–Robertson–Walker (FRW) universe undergoing a ‘‘bounce’’ and ask what the absolute minimum requirements are for such a bounce to occur. Not too surprisingly, the strong energy condition (SEC) of classical gravity must be violated [7–9]. (SEC–violation is a necessary but not sufficient condition.) More surprisingly, for universes with positive spatial curvature, *none* of the other energy conditions *need* be violated. We shall present a model-independent analysis of the bounce similar to the model-independent analysis applied to the Morris–Thorne traversable wormholes [10–15], and also show with specific examples how the various cosmological singularity theorems [3,4] and their modern extensions [16–20] can be evaded. Finally we discuss the extent to which SEC violations are compatible with known physics, and exhibit an infla-

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tion-inspired model for which a big-bounce is ''natural''.

A bouncing baby universe, or Tolman wormhole, is simply a FRW universe that undergoes a collapse, instant of maximum compression, and subsequent expansion (as opposed to undergoing a big crunch singularity or exhibiting a big-bang singularity). In a model-independent analysis, the key idea is to extract as much information as possible from the energy conditions without making any particular commitment to the equation of state for the matter content of the universe [10–15]. The utility of such an approach has recently been demonstrated in a different context: applying the energy conditions to the epoch of galaxy formation [21–23].

The FRW cosmology is described by the metric [3–6]

$$ds^2 = -dt^2 + a(t)^2 \times \left[\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right], \tag{1}$$

with $k = +1$, 0, or -1 for hyperspherical, flat, or hyperbolic spatial sections, respectively.

To have a bounce, there must be some time at which the size of the universe is a minimum. We take this to be time zero, and use a subscript $*$ to denote quantities evaluated at the bounce $t=0$:

$$\dot{a}_* = 0; \quad \ddot{a}_* \geq 0. \tag{2}$$

This weak inequality on $\ddot{a}_*$ is not enough for proving our deeper results. Since we want time zero to be a true *minimum*, we must have $a > a_*$ for $t \neq 0$, with this now being a strict inequality. An application of the fundamental theorem of differential calculus now implies:

$$\exists \tilde{t} > 0 : \forall t \in (-\tilde{t}, 0) \cup (0, \tilde{t}) \quad \ddot{a} > 0. \tag{3}$$

This is the analog, for a bouncing baby universe, of the Morris–Thorne ''*flare-out*'' condition for traversable wormholes [10]. (See also [11] (Eq. (11.12), p. 104) and [12–15].) *Mutatis mutandis*, there will be similar open regions surrounding the bounce for which

$$\frac{\mathrm{d}^2[\ln(a)]}{\mathrm{d}t^2} > 0; \quad \text{and} \quad \frac{\mathrm{d}^2(a^3)}{\mathrm{d}t^2} > 0, \tag{4}$$

with these again being strict inequalities.

For a FRW universe the Einstein equations reduce to

$$\rho = \frac{3}{8\pi G} \left[\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right], \tag{5}$$

$$p = -\frac{1}{8\pi G} \left[2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right]. \tag{6}$$

Some quantities of interest are,

$$\rho + p = \frac{1}{4\pi G} \left[-\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right] = \frac{1}{4\pi G} \left[-\frac{\mathrm{d}^2}{\mathrm{d}t^2} \ln a + \frac{k}{a^2} \right], \tag{7}$$

$$\rho - p = \frac{1}{4\pi G} \left[\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2} \right] = \frac{1}{4\pi G} \left[\frac{1}{3a^3} \frac{\mathrm{d}^2(a^3)}{\mathrm{d}t^2} + 2\frac{k}{a^2} \right], \tag{8}$$

and

$$\rho + 3p = -\frac{3}{4\pi G} \left[\frac{\ddot{a}}{a} \right]. \tag{9}$$

By the strict inequalities discussed above [Eqs. (3)–(4)], there will be open temporal regions surrounding the bounce for which

$$\rho + p < \frac{1}{4\pi G} \left[\frac{k}{a^2} \right]. \tag{10}$$

$$\rho - p > \frac{1}{4\pi G} \left[\frac{k}{a^2} \right]. \tag{11}$$

$$\rho + 3p < 0. \tag{12}$$

For comparison, the standard point-wise energy conditions are the *null energy condition* (NEC), *weak energy condition* (WEC), *strong energy condition* (SEC), and *dominant energy condition* (DEC). Their specializations to a FRW universe have previously been discussed in Refs. [21–23]. Basic definitions are given in Refs. [3,11].

$$\text{NEC} \Leftrightarrow \quad (\rho + p \geq 0). \tag{13}$$

$$\text{WEC} \Leftrightarrow \quad (\rho \geq 0) \text{ and } (\rho + p \geq 0). \tag{14}$$

$$\text{SEC} \Leftrightarrow \quad (\rho + 3p \geq 0) \text{ and } (\rho + p \geq 0). \tag{15}$$

$$\text{DEC} \Leftrightarrow \quad (\rho \geq 0) \text{ and } (\rho \pm p \geq 0). \tag{16}$$

As applied to a bouncing baby universe we have: First, by working at the bounce itself [where we only have the weak inequality (2)]

$$\exists\,\text{Bounce} + [k=-1] \Rightarrow \quad \text{NEC violated}, \tag{17}$$

$$\exists\,\text{Bounce} + [k=0;\ \ddot{a}_* > 0] \Rightarrow \quad \text{NEC violated}, \tag{18}$$

$$\exists\,\text{Bounce} + [k=+1;\ \ddot{a}_* > a_*^{-1}] \Rightarrow \quad \text{NEC violated}. \tag{19}$$

(In particular, any of these three conditions automatically implies violation of the WEC, SEC, and DEC.) Thus a bounce in a hyperbolic $(k=-1)$ universe must violate *all* the pointwise energy conditions, a bounce in a spatially flat $(k=0)$ universe is on the verge of violating *all* the energy conditions, and a sufficiently gentle bounce in a hyperspherical $(k=+1)$ universe exhibits a ‘‘window of opportunity’’ that requires more detailed analysis. Secondly, by working in suitable open regions surrounding the bounce [and using the strict inequalities (3)–(4) derived above] we obtain the stronger results

$$\exists\,\text{Bounce} + [k \neq +1] \Rightarrow \quad \text{NEC violated}, \tag{20}$$

$$\exists\,\text{Bounce} \Rightarrow \quad \text{SEC violated}. \tag{21}$$

Thus the energy condition violations are minimized by taking the universe to be hyperspherical $(k=+1)$ and by making the bounce sufficiently gentle: $\ddot{a}_* \leq a_*^{-1}$. In this case it is easy to check that NEC, WEC, and DEC are satisfied, and *only* SEC need be violated. Indeed we only need SEC to be violated in some open temporal region surrounding the bounce, and it is quite possible to satisfy *all* the point-wise energy conditions at sufficiently early and late times:

$$\exists\,\text{Bounce} + [k=+1;\ \ddot{a}_* \leq a_*^{-1}] \Rightarrow \quad \text{NEC, WEC, DEC satisfied; SEC violated}. \tag{22}$$

This is to be contrasted to the situation for Morris–Thorne traversable wormholes, wherein (for spherically symmetric wormholes) there is an open spatial region surrounding the throat over which the NEC (and therefore also the WEC, SEC, and DEC) must be violated [10–13]. Generalization of all these results to spacetimes more general than the FRW universes, along the lines of [14,15] is certainly possible, and we intend to address this issue more fully in a subsequent paper [24].

A simple specific example of a geometry that satisfies NEC, WEC, and DEC but violates SEC is

$$ds^2 = -dt^2 + \left(a_*^2 + \beta^2 t^2\right) \times \left[\frac{dr^2}{1-r^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right], \tag{23}$$

provided we take $\beta < 1$. Note that this is the temporal analog of the toy model traversable wormhole considered on page 398 of the Morris–Thorne article [10]. Explicit calculation of the stress-energy components yields

$$\rho = \frac{3}{8\pi G}\,\frac{a_*^2 + \beta^2[1+\beta^2]t^2}{\left(a_*^2+\beta^2 t^2\right)^2} > 0. \tag{24}$$

$$p = \frac{-1}{8\pi G}\,\frac{a_*^2[1+2\beta^2] + \beta^2[1+\beta^2]t^2}{\left(a_*^2+\beta^2 t^2\right)^2} < 0. \tag{25}$$

So that

$$\rho + p = \frac{2}{8\pi G}\,\frac{a_*^2[1-\beta^2] + \beta^2[1+\beta^2]t^2}{\left(a_*^2+\beta^2 t^2\right)^2} > 0. \tag{26}$$

$$\rho - p = \frac{2}{8\pi G}\,\frac{a_*^2[2+\beta^2] + 2\beta^2[1+\beta^2]t^2}{\left(a_*^2+\beta^2 t^2\right)^2} > 0. \tag{27}$$

$$\rho + 3p = \frac{-6}{8\pi G}\,\frac{a_*^2\,\beta^2}{\left(a_*^2+\beta^2 t^2\right)^2} < 0. \tag{28}$$

The condition $\beta < 1$ is used to keep $\rho + p$ positive definite and prevent violations of the NEC. The pressure is not positive in this toy model, (nor does it need to be positive to satisfy the energy conditions).

A second specific example of a geometry that also satisfies NEC, WEC, and DEC but violates SEC is

$$ds^2 = -dt^2 + a_*^2 \cosh^2(Ht) \times \left[\frac{dr^2}{1-r^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right], \tag{29}$$

provided we take $H < a_*^{-1}$. Note that this is *not* de Sitter space, since de Sitter space would correspond to $H \equiv a_*^{-1}$. Explicit calculation of the stress-energy components yields

$$\rho = \frac{1}{8\pi G}\left(3H^2 + \frac{3(1-a_*^2 H^2)}{a_*^2 \cosh^2(Ht)}\right) > 0. \tag{30}$$

$$p = \frac{-1}{8\pi G}\left(3H^2 + \frac{(1-a_*^2 H^2)}{a_*^2 \cosh^2(Ht)}\right) < 0. \tag{31}$$

So that

$$\rho + p = \frac{1}{8\pi G}\left(\frac{2(1-a_*^2 H^2)}{a_*^2 \cosh^2(Ht)}\right) \geq 0. \tag{32}$$

$$\rho - p = \frac{1}{8\pi G}\left(6H^2 + \frac{4(1-a_*^2 H^2)}{a_*^2 \cosh^2(Ht)}\right) > 0. \tag{33}$$

$$\rho + 3p = \frac{-6}{8\pi G} H^2 < 0. \tag{34}$$

The condition $H < a_*^{-1}$ is now used to keep $\rho + p$ positive and prevent violations of the NEC. The pressure is again not positive in this toy model, nor does it need to be positive to satisfy the energy conditions.

Perhaps the best known cosmological singularity theorems are the Penrose-Hawking and Geroch theorems [3,4]. Both of these theorems explicitly use the SEC as an input hypothesis, so violating the SEC vitiates these theorems. Now when it comes to the singularity theorems relevant to black hole formation, it was rapidly realized that the original black hole singularity theorem (which also uses the SEC [3,4]) could be modified to produce more powerful theorems that used weaker energy conditions (e.g, the NEC [3,4]). It was commonly believed (in at least some circles) that the cosmological singularity theorems could be similarly strengthened, and there are in fact a number of newer cosmological singularity theorems that use the NEC (but at the cost of adding other rather strong conditions) [16–20]. How does the present discussion evade the consequences of these theorems?

The theorems of [16,17] are stated using the WEC but really only need the NEC. However the key assumption made there is that the universe is open in the mathematical sense (which in a FRW universe implies the universe is either hyperbolic, $k = -1$, or flat, $k = 0$). Thus these singularity theorems are compatible with the results of this Letter since we have explicitly shown that a bounce in hyperbolic or flat FRW universes requires NEC violation.

The closed universe singularity theorem of [18] uses a very strong technical requirement (compact localized past light cones) explicitly violated by our models. The more recent results in Refs. [19,20] are also compatible with our results in that the theorems either apply to open universes, or make additional technical assumptions violated by our analysis.

Physical reasonableness of the SEC: It is relatively difficult to violate the NEC, WEC, and DEC; violations of these energy conditions typically being due to (small) quantum effects. On the other hand, it is rather easy to violate the SEC, leading some researchers to refer to the SEC as ‘‘the unphysical energy condition’’. Violations of the SEC are generic to classical scalar fields [3,11], to inflationary spacetimes [16–20], to spacetimes with (cosmologically large) positive cosmological constants [21–23], to certain mean-field quantum field theories [25,26] and other quantum mechanical situations [8], and significantly, to classical relativistic fluids with two body interactions [9,27]. A particularly wide class of Tolman wormholes can also be constructed by Wick rotating Euclidean wormholes back to Lorentzian signature [28]; the Wick rotated Euclidean wormholes typically satisfying all energy conditions except the SEC. These observations are important in that they elevate the discussion of this Letter from a mere mathematical curiosity to an issue that merits serious attention.

An inflation-inspired model: The generic feature common to all inflationary FRW models is the introduction of a minimally coupled scalar field ϕ called the inflaton (in addition to whatever matter is normally present). The inflaton contributes to the energy density and pressure:

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi), \tag{35}$$

$$p_\phi = \tfrac{1}{2}\dot{\phi}^2 - V(\phi), \tag{36}$$

and the inflaton field satisfies the equation of motion

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0. \tag{37}$$

The key feature relevant for the present discussion is that

$$\rho_\phi + 3p_\phi = 2\dot{\phi}^2 - 2V(\phi). \tag{38}$$

If the inflaton field bounces at the same time as the geometry (i.e., $\dot{\phi}_* = 0$), then the inflaton field can be used as the natural candidate for providing the SEC violations required to support the bounce:

$$(\rho + 3p)_{\text{total}} = (\rho + 3p)_{\text{normal}} + 2\dot{\phi}^2 - 2V(\phi). \tag{39}$$

$$(\rho + p)_{\text{total}} = (\rho + p)_{\text{normal}} + \dot{\phi}^2. \tag{40}$$

$$(\rho - p)_{\text{total}} = (\rho - p)_{\text{normal}} + 2V(\phi). \tag{41}$$

Thus adding a spatially-homogeneous inflaton field to normal (energy condition satisfying) matter preserves the NEC, WEC, and DEC, but can easily lead to violations of the SEC. In this sense inflation (either old or new inflation; but not chaotic or eternal inflation) is naturally compatible with the bounce scenario. With typical estimates for the inflaton VEV being of order the GUT energy scale, we would similarly estimate the bounce to occur when the radius of the universe is about one GUT distance scale (about 1 000 Planck lengths). This is certainly a small distance, even by particle physics standards, but because it is so much larger than the Planck scale, we may still reasonably hope for the applicability of semiclassical quantum gravity — thus we now hold out the reasonable hope for a big bounce that not only evades the classical singularity theorems but also evades the necessity for dealing with the full theory of quantum gravity.

In summary, (1) replacing the big bang with a big bounce violates the SEC but does not necessarily violate any of the other energy conditions, and (2) violating the SEC is relatively easy and can be achieved at the classical level, without needing to appeal to quantum effects. Of course, even more exotic variations can be contemplated. You can consider the effects of violating *all* the energy conditions [29], including the NEC, or even more boldly you can consider having the universe bootstrap itself into existence via a chronology violating region [30]. A key aspect of this Letter is that extreme steps of this type are not necessary: the big bang singularity can be tamed with relatively mild modifications of the standard cosmological model. Indeed, there are simple extensions of either old or new inflation for which such a bounce is ''natural''.

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