Annals of Pure and Applied Logic 48 (1990) 299–301
North-Holland

CORRIGENDUM

CORRECTION TO "UNDECIDABILITY OF $L(F_\infty)$ AND OTHER LATTICES OF R.E. SUBSTRUCTURES"

Rod DOWNEY
Department of Mathematics, Victoria University of Wellington, P.O. Box 600, Wellington, New Zealand

Communicated by A. Nerode
Received 16 January 1990

In [1], the author gave a proof that a wide class of lattices of r.e. substructures had undecidable first-order theories. The proof consisted of two parts. First an r.e. set A was constructed so that A had the following two properties:

(i) A had the *lifting property* (see below).

(ii) A had an r.e. superset B such that the lattice $L^*(A, B)$ of r.e. sets containing A and contained in B (modulo finite sets) was effectively isomorphic to $\mathscr{E}^*$, the lattice of r.e. sets modulo finite sets.

The second part of the proof was a purely algebraic argument that showed such sets A and B could be used to code enough of $\mathscr{E}^*$ into the relevant lattices to establish the undecidability of these lattices.

Unfortunately the proof (in [1]) of the first part contained an error. Specifically the set A constructed in [1] fails to have property (ii) for *all* r.e. supersets B of A. The error occurs in Lemma 3.5 where the proof only shows that there is an embedding of $\mathscr{E}^*$ into $L^*(A, B)$.

We repair this flaw here.

That is, we construct an r.e. set A with properties (i) and (ii) above (with $B = \omega$). We recall the following definition from [1].

Definition. An r.e. set A has the *lifting property* if A is coinfinite and for all r.e. strong arrays $\{D_{g(x)}: x \in \omega\}$, for almost all x, $|D_{g(x)} - A| \leq 1$.

To repair the proof from [1], we prove the next theorem.

Theorem. *There exists an r.e. set A with the lifting property such that $\bar{A}$ is semilow* (i.e., $\{x: W_x \cap \bar{A} \neq \emptyset\} \leq_T \emptyset'$).

By the results of Soare [3] (cf. [2]), if A is r.e. and $\bar{A}$ is semilow, then $L^*(A, \omega)$ is effectively isomorphic to $\mathscr{E}^*$. From this we get both properties (i) and (ii) for A.

To prove the theorem, we build $A = \bigcup_s A_s$ in stages. At each stage s, we let $\{a_{i,s}: i \in \omega\}$ list in order $\bar{A}_s$. We shall meet the requirements below.

R_e: If $\{D_{\varphi_e(x)}: x \in \omega\}$ is a strong array, then $(aax)(|D_{g(x)} - A| \leqslant 1)$,

N_e: $\lim_s a_{e,s} = a_e$ exists,

Q_e: $(\exists^\infty s)(W_{e,s} \cap \bar{A}_s \neq \emptyset) \Rightarrow (W_e \cap \bar{A} \neq \emptyset)$.

Here φ_e denotes the eth partial recursive unary function and W_e denotes the eth r.e. set. Since we need only meet R_e just in the case that $\{D_{\varphi_e(x)}: x \in \omega\}$ is a strong array we adopt the convention that for all e, x, y and s if $\varphi_{e,s}(s)\downarrow$ and $\varphi_{e,s}(y)\downarrow$, then $D_{\varphi_e(x)} \cap D_{\varphi_e(y)} = \emptyset$. To do this we need only control the enumeration of $\varphi_{e,s}$. Note that meeting Q_e makes $\bar{A}$ semilow by the limit lemma.

For the sake of Q_e we define *associates* $\Lambda(e, s)$ which will be members of $\bar{A}_s$. At any stage s either $\Lambda(e, s) = a_{j,s}$ for some j or $\Lambda(e, s)$ is undefined. We say that Q_e requires attention at stage $s + 1$ if $\Lambda(e, s)$ is undefined and $(\exists y)(y \in W_{e,s} \cap \bar{A}_s$ and $y = a_{j,s+1}$ for some $j > e)$. For the least such y we say that Q_e requires attention *via* y. We say that Q_e is *injured* at stage $s+1$ if $\Lambda(e, s)$ is defined and $\Lambda(e, s) \in A_{s+1}$ (this automatically makes $\Lambda(e, s)$ undefined).

Construction

Stage 0. Set $A_0 = \emptyset$ and declare $\Lambda(i, 0)\uparrow$ for all i.

Stage $s + 1$.

Step 1 (Meeting R_e). Perform the following substages $e \leqslant s$ in order of e. Append a superscript e to a parameter to denote its stage at the end of substage e.

Substage 0. Set $a^0_{i,s+1} = a_{i,s}$ and $\Lambda^0(j, s+1) = \Lambda(j, s)$ for all i, j.

Substage $e + 1$ $(e \leqslant s)$. For each $x \leqslant s$ if $\varphi_{e,s}(x)\downarrow$ and

$$D_{\varphi_e(x)} \cap (\{a^e_{0,s+1}, \ldots, a^e_{e,s+1}\} \cup \{a^e_{j,s+1}: a^e_{j,s+1} \leqslant \max\{\Lambda^e(j, s+1): j \leqslant e\}\}) = \emptyset, \quad (1)$$

if $|D_{\varphi_e(x)} \cap \bar{A}^e_{s+1}| \geqslant 2$, we enumerate all except one of the members of $D_{\varphi_e(x)} \cap \bar{A}^e_{s+1}$ into A^{e+1}_{s+1} according to the following rule. If $D_{\varphi_e(x)} \cap \bar{A}^e_{s+1}$ contains some $\Lambda^e(j, s+1) = a^{e+1}_{k,s}$ for $k > j$ choose the least such j to keep $\Lambda^e(j, s+1)$ out of A^{e+1}_{s+1}, and enumerate the remainder into A^{e+1}_{s+1}. If $D_{\varphi_e(x)} \cap \bar{A}^e_{s+1}$ does not contain any such $\Lambda^e(j, s+1)$ keep the least member of $D_{\varphi_e(x)} \cap \bar{A}^e_{s+1}$ in $\bar{A}^{e+1}_{s+1}$.

Step 2 (Meeting Q_e). Let $a^{s+1}_{i,s+1} = a_{i,s+1}$ (for all $i \in \omega$). Find the least e if any such that Q_e requires attention. If e exists, let y be least for e and define $\Lambda(e, s+1) = y$. □ End of construction

To verify the construction above, we argue by simultaneous induction that $\lim_s a_{e,s} = a_e$ exists and $\lim_s \Lambda(i, s) = \Lambda(i)$ exists. From this (1) will give finite restraint in the limit, and hence Step 1 will ensure that $(aax)(D_{\varphi_e(x)} \cap \bar{A} \leqslant 1)$. For an induction, suppose that $a_{e,s_0} = a_e$ and for all $j \leqslant e$, $\Lambda(e, s_0) = \Lambda(e)$. Now $a_{e+1,s}$ can only change at stages $s > s_0$ due to the action of R_j for $j \leqslant e$. Since $|D_{\varphi_i(x)}| < \infty$, we can also suppose at s_0 that for all $j \leqslant e$ and all x if $D_{\varphi_j(x)} \cap \{a_j: j \leqslant e\} \neq \emptyset$, then

D, $D_{\varphi_{j,s_0-1}}(x)\downarrow$. Since, in Step 2 the default is to leave the least $a_{j,s}$ threatened, it follows that after s_0, $a_{e+1,s}$ can only change to become a $\Lambda(k, s)$. Thereafter $a_{e+1,s}$ can only change to become $\Lambda(k', s)$ for $k' < k$. It follows that $\lim_s a_{e+1,s} = a_{e+1}$ exists, and similarly $\lim_s \Lambda(j, s) = \Lambda(j)$ exists. □

References

[1] R.G. Downey, Undecidability of $L(F_\infty)$ and other lattices of r.e. substructures, Ann. Pure Appl. Logic 32 (1) (1986) 17–26.

[2] W. Maass, Characterization of recursively enumerable sets with supersets effectively isomorphic to all recursively enumerable sets, Trans. Amer. Math. Soc. 279 (1983) 311–336.

[3] R.I. Soare, Automorphisms of the lattice of recursively enumerable sets, Part II: low sets, Ann. Math. Logic 22 (1982) 69–107.