Maths and Scottish Country Dancing

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- This is a modified version of what I did for a talk for the RSNZ Napier Branch, as a public lecture.
- During the lecture I was using videos, and in this version, I will put the links.
- For best results, you can look at the relevant links.

Que?

- I will try to explore what I see as common ideas I have when I think about some parts of mathematics and think about Scottish country dancing.
- These involve
  1. Combinatorics
  2. Flow in space
  3. Aesthetics
- Hopefully this will give some insight into what we do in modern mathematics, and convince you to start Scottish country dancing.

Mathematics

"The book of nature is writ in the language of mathematics."-Galileo

Plan

- So what do mathematicians do?
- Where did it come from?
- The backbone of modern society.
- Miscellaneous examples.
- A couple of pointers as to what kinds of stuff I do.

Research

- What research is useful/important?
- It is pretty clear that it is hard for even the experts to anticipate what will prove to be important.
- We see a couple of examples in this talk.
- I realize that most research is “targeted” for outcomes that are easily seen to be important and practical.
- Here in New Zealand, for instance....

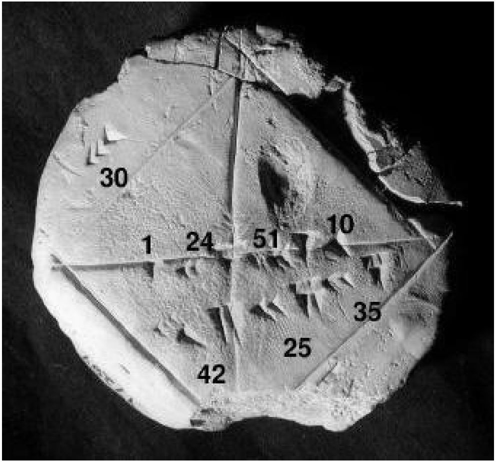
Mathematics

- So what do we do?
- Ultimately I think mathematicians build symbolic models of the world.
- Then manipulating them allows them to understand/predict/explore.
- The Egyptians/Babylonians/Greeks/Chinese/Incas invented geometry to help building and the motions of the cosmos etc.
- They and others invented methods of calculating interest rates etc to make money.
- Later from physics we invented differential equations which can be used to describe rates of change. Witness the CT scan above.
- Combinations of these methods together with statistics and graph theory used to model COVID-19.

Cuneiform tablet

1900 BCE

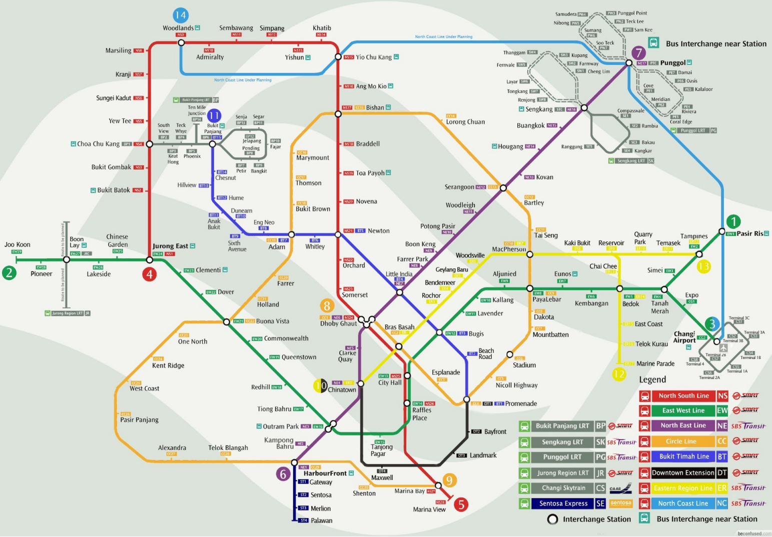
$$\sqrt{2} = 1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \approx 1.414213.$$

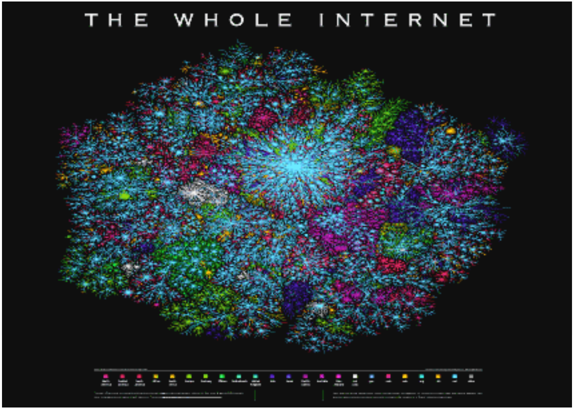


An Combinatorial Example

- From my own work.
- Graphs: These are abstract models consisting of points ("vertices") and lines ("edges") between them.
- Graphs are abstract models of many situations.
- For example, the points might be people and we might connect if they are friends.
- Maybe we might then "cluster edit" to find groups of mutual friends cliques.
- This is a high level view of how ad targeting in e.g. Google works.
- But the points might represent bits of DNA and we might be figuring out what causes a disease, etc.
- Might be a cluster of people who might have exposure.
- Or it might be bits of music and measuring similarity using "Kolmogorov complexity"

Some examples




THE WHOLE INTERNET

Algorithms

- What is the point of this abstraction? The thing is that if we understand properties of types of graphs, then no matter what the application the properties will hold.
- For example, diffusion analysis was originally used to model flow of a liquid but similar analysis can be used for spread of virus.
- In my work, this abstraction idea has applied to algorithm design.
- We gave an approach which gave methods (algorithms) for, e.g. cluster editing in certain kinds of graphs.
- Algorithms here mean sequences of instructions which tell you how to do something.
- Baking a cake; working out your tax return; assembling flat furniture.

Parameterized algorithmics

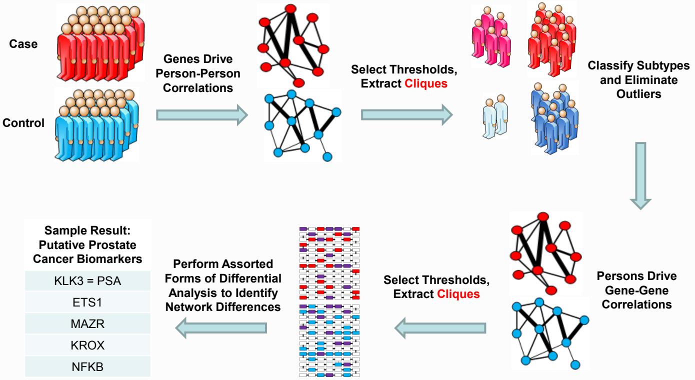
- Fellows and I initiated an area called Parameterized Algorithmics where the computational complexity and the algorithmics are fine grained around the many parameters a problem might have. Before that problems were considered by size alone.
- Interestingly, my work with Fellows was based around "well quasi-ordering theory" of finite graphs.
- It was pure blue skies research.
- Once people were sensitized to the ideas of using fine-grained multi-parameter approach, we found:
- A plethora of new sensitive approaches to systematic algorithm design.
- Many new algorithms.
- Near optimal algorithms.

Applications of my own work

- Mike Langston and his University of Tennessee team.
- Prostate Cancer

APAC 2012

Application: Prostate Cancer

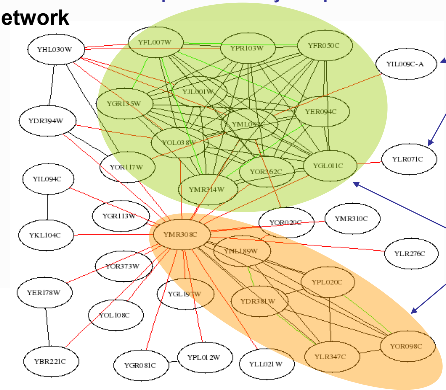


APAC 2012

Application, Protein Complex Prediction

Protein-Protein Interaction Network

Peptidase activity complex



yeast proteins

protein complexes

edge deleted

edge added

Protein binding complex

Recognize as Cluster Edit

Applications of my own work

- Peter Shaw, Faisal N. Abu-Khzam, Robyn Marsh, Heidi Smith-Vaughan
- Otitis Media (an ear infection) Northern Territory, Australia.
- 30% of aboriginal children are deaf
- 97.5% (!) of indigenous inmates.
- Not understood multi-pathogen disease, so Network Analysis.
- Multi-variable (parameterized) analysis combined with traditional statistical methods (which alone failed).

Online Algorithmics-New obsession

- Much of my work deals with algorithms in a hostile environment. Reactive algorithms working with incomplete information.
- BIN PACKING. We have a set of bins $b_1, b_2, \ldots$ and need to pack a stream of objects, $o_1, \ldots, o_m$ given. If you see the objects and bins at the beginning, then the problem is offline. If I give you the objects as a stream, then it is online. Think about packing a car boot.
- You are a triage nurse and need to schedule patients to be seen.
- You are scheduler (maybe in a computer) and need to assign memory or some other resource to users.
- The Opponent typically is a hostile universe trying to give you the worst sequence of objects, requests, etc to cause you to do badly as compared with the offline version. (Or maybe there is no offline version.)

Online

- In all of the above you are in an online situation, though some are different than others.
- There are hundreds of algorithms for such problems both in the finite case, and in the case where e.g. a scheduler needs to works "forever".
- There are books with taxonomies of such algorithms.
- My current goal is to give a theoretical basis for the above.
- What does a theoretical basis do for you? Before my work with Fellows, (in retrospect) there were parameterized algorithms, but our explicit basis allowed for systematic development. We became sensitized to the issues.
- My online builds on my earlier work on computability theory, which seeks to understand computation.

Online colouring

- We want to online colour a tree (or a forest).
- I claim the universe can force us to use log n many colours for a tree of size n. Offline only needs 2 colours.
- We can think of as a game. Player I must colour a vertex and then II gives the next one and all the connections.
- A proof of this requires a showing that Player II who is providing the bad tree has a dynamic strategy, whatever I plays.
- This involves a visualization of how the game is unfolding with time.

- I give you one vertex, colour 1.
- I give you another joined to the first, you colour 2. 1—2.
- I give you a third you must choose : 1 or 2. You choose 1. 1–2 1. If you choose 2, I would regard this as 2 1–2.
- I give you another v. If you chose 2 I join to the first vertex and the 2, if you chose 1, the I join to the original 2 and the new 1. e.g 1–2–v–1 (or 2–v–1 2)
  You must choose a new colour.
- Now this means that with 4 vertices you can force 3 colours.
- Now repeat this twice more so you have 3 lines 4 vertices using 3 colours.
  1-2-3-1
  2-3-1-2
  1-2-3-1
- Now add one more w and connect w to colours 1, 2, 3, one from each line. Now a 4th colour must be needed, etc.

Bin Packing

- For example, we have bins of size 10. Hostile opponent gives us a packet of size 5, and we put in bin 1. She gives us next packet of size 2. If we use first fit we put in bin 1. But the next one might be size 5 and would have been cleverer to have put it into bin 1 had we but known....
- FIST FIT for online bin packing gives a 2-approximation ratio.
- But is there a better approximation algorithm? If not how to show that? How to find the best algorithm?
- What do we mean by that anyway?
- For BIN PACKING the current best algorithm in terms of performance ratio has ratio 1.58889 guarantee.

Priority Arguments

- A lot of my work involves "priority arguments" in which we build an infinite object a little bit at a time balancing the competing wishes of infinite teams each competing for properties of the objects.
- One team is antithetical to the other, but all wishes must eventually be satisfied.
- For example, in a simple case, a team of industrialists (wanting to build factories) and a team of environmentalists (wanting to build parks).
- Like a boat navigating a safe path in a sea of hostility.

Summary

- So in this kind of mathematics we do a kind of thought experiment, imagining how the hostile opponent might work in time and develop a strategy which copes with this.
- The key aspect of all of this is the visualisation of what is happening to objects as they evolve with time. From "The Chess Mind", by Gerald Abrahams:

  > *"the capacity of the mind for making a path through time and complexity, is the essence and moving edge of any intellectual process."*

- Then we need to develop mathematics to show that what we are doing works.
- And then show it is relevant to practice.

- What are the similarities between the problems? Why do they have algorithms with constant performance ratios? etc.
- My current project is to understand using logic what kinds of problems exhibit constant approximation ratios and try to give a high level description.

> *A problem with a high level description of "type X" for a collection of problems of "type Y" will have a good approximation algorithm, and here is a way of generating this algorithm, from the description X and Y alone.*

Scottish Country Dancing

- What is it?
- It is not Highland Dancing, which is competitive, mainly individual, and done by young people. It is social dancing.
- It is derived from traditional dancing as performed since the 18th Century in Scotland, and was formed in its present form in 1923.
- By two very strong Scotswomen, Mrs Ysobel Stewart and Dr. Jean Milligan.
- It is done in teams ("sets") of usually 4 couples, and involves combinations of "formations".
- They are typically 2-, 3-, or 4-couple dances within the set and people progress up and down through the set in what is called a progression.
- For example, if the dance is 2-couple then the progress would be

$$1234 \to 2134 \to (23)(14) \to 3241 \to (34)(21) \to 4312 \to \dots$$

An example

- A huge tradition of social dancing.
- Now all around the world.
- The De'il Amang the Tailors is an old dance which is loved worldwide, for the choreography and music combination.
- This is a three couple dance in a four couple set.
- In the lecture I played:
  `https://www.youtube.com/watch?v=vkXC9shBnYk`

Devising and teaching

- The music is king.
- A living tradition, going back to the 18th Century and earlier, but lots of modern music.
- SCD has quicktime and slow music.
- The quicktime is either a reel or a jig (2/4,4/4, or 6/8).
- The slowtime is called a Strathspey (although sometimes an air or pastoral which are beautiful but not traditional) and is either 2/4 or 4/4.
- Here are some examples.
- I used some that are not online, but `http://www.johnsonvillescd.org.nz/resources/practicing-dancing/` has a bunch of music to listen to.

- For me, the single most important part of the dance is the interpretation of the music.
- (You might think that this is a characteristic of all dance, but e.g. the contemporary dance choreographer Merce Cunningham famously did not. Maybe he'd never heard a stirring Scottish Reel.)
- Where should we be and when?
- How should we try to modify our phrasing to enable the dance for ourselves and others?
- One simple example is a formation called a reel.
- I used the Right Shoulder Reels from `http://www.lowerhuttscd.org.nz/index.php/teaching-tools/`

Phrasing

- Here's part of a dance devised by Hugh Foss (who lead the team which broke the Japanese Enigma) called The Celtic Brooch
- It shows clearly how the team phrasing works.
- I used: `https://www.youtube.com/watch?v=P5Md8qZPCo4&feature=emb_logo`

One of mine

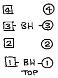
- This is called The Twisted Wizard of Binghamton
- I will show you the diagrams but as I have no video, I will show the video of another of mine called My Golden Bear I wrote for Kristin. The central figure called "Corners Pass and Turn in (lead change) Tandem" is mine.
- I used `https://www.youtube.com/watch?v=DrlXR-vaTts&feature=emb_logo`

The Tuatara Collection
Rod Downey

THE TWISTED WIZARD OF BINGHAMTON

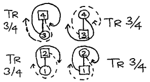
40 Strathspey

② 1-2



1 & 3C X BH in passing face down

② 3-4



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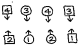
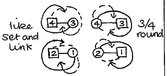


② 5-6

All give hands (SET)

② 7-8

All overlink in pairs



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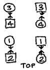
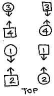


2 & 4 C's face down

⑧ 9-16

Reel of 4 on side finish
2's facing 1's
4's facing 3's



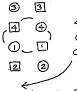
② 17-18

All set on sides
2's with 1's
4's with 3's

② 19-20

Giving hands
All set on sides

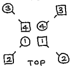
④ 21-24



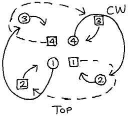
4 & 1 C's circle L once round

pull back Rsh on bar 24 to face corner

to

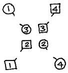


④ 25-28



while 2,3 C's pass others Rsh
circle L once round
pull Rsh to face corner pos.

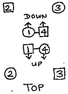
to



④ 29-32

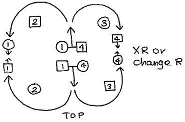
repeat 25-28 with roles reversed from progressed positions

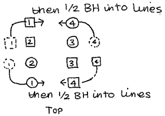
to



finish near hands joined

⑥
33-38

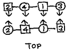




- IW & 4M dance out through top

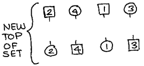
  IM & 4W dance out through bottom
- IC cast on W's side
  4C cast on M's side
- IC & 4C XR with partner on sides continue going way going
- IC & 4C begin to dance in across the set,
  IM & IW above 1st couples place
  IW & 4M at bottom below 4th couple's place
- IM & 4W and IW & 4M 1/2 BH turn to finish in 2 lines across

to



②
39-40
All SET in lines giving hands

Start again
with set 90° rotated CW



Repeat 2 more time
each set rotates CW
90° to original places.

Aesthetics

- I can find both maths and SCD quite beautiful.
- It is sometime hard to say why we find a piece of music so lovely, or a painting, or why a formation or dance appeals to my sense of beauty.
- All I can offer are examples.

A beautiful piece of maths

Theorem (Euler, 1752 (kind of))

For a connected plane graph G, Euler's Formula: For $E \geq 1$, and G connected. $V + E - F = 2$

V =the number of vertices, F=the number of faces and E= the number of edges.

Actually Euler's original proof was incorrect, and a version of this result was stated by Descartes in 1650. The first complete proof was by Legendre in 1794.

Planar Graphs and Maps

G is called plane if it is drawn on the plane with no edges crossing.
Maps can be converted into planar graphs.

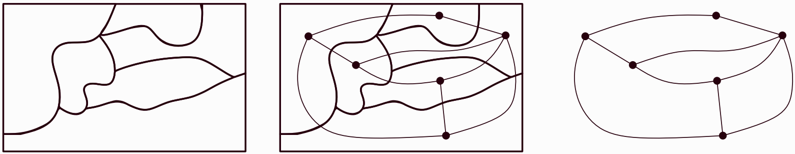


Figure: Translating a map into a plane graph.

A colouring of the map where no bordering countries have the same colour corresponds to a colouring of the vertices where no two adjacent vertices have the same colour.
This has 6 vertices(V), 9 edges (E), and 5 faces (F). $6 + 5 - 2 = 9$.

The Raging Sea

- Your plane graph is sitting inside a raging sea, breaking on the structure from the outside.
- You remove an outer edge and the sea floods in. : $F = F - 1$, $E := E - 1$, $V := V$. Note we remain connected.
- Continue till only a jetty remains. Then remove outer edges one at a time: $E := E - 1$, $V : V - 1$, $F = 1$ still.
- At the end $V + E - F$ has not changed, but you have a single edge with 2 vertices, and one face. so $V + E - F = 2 + 1 - 1 = 2$.

Why is is beautiful?

- First the statement tells us something universal. All plane graphs have this property.
- It is unexpected
- The proof is *intuitive, visual and simple*. With the stroke of a few lines we see something eternal about all plane graphs.
- Erdös would haver said it was a proof from the book.

Another beautiful proof

Theorem (in Euclid-300BCE, but likely Hippasus of Metapontum or Theodorus-500BC)

$\sqrt{2}$ is irrational. That is, there are no integers p, q with $\sqrt{2} = \frac{p}{q}$.

[Courant and Robbins] " This revelation was a scientific event of the highest importance. Quite possibly it marked the origin of what we consider the specifically Greek contribution to rigorous procedure in mathematics. Certainly it has profoundly affected mathematics and philosophy from the time of the Greeks to the present day. "

Pythagoreans

- This was actually a quasi-religious cult.
- It is certainly thought that Hippasus's proof was suppressed.
- There are stories about him being executed for the heresy. (He sleeps with the fishes.)
- The cat came out of the bag with the proof in Euclid's Elements, one of the greatest achievements of intellectual thinking in history. (This took at least 100 years.)

The Elements

> *"The findings of over two hundred years of Greek geometry and number theory, and fifteen hundred years of Babylonian mathematics, were rigorously established from first principles. Starting from a handful of simple statements, and proceeding exorably one small step at a time, Euclid obtains result after result of genuine depth. Achievements wrung over the ages with great difficulty were made to seem inevitable."*
> *....Two thousand years of geometry in a few short compelling pages"'*
> *Donald O'Shea "The Poincaré Conjecture"*

Notably the third most read book in history after the Bible and the Q'ran.
Used for 2000 years to train minds.

A proof from Euclid

- Proof by contradiction. Suppose that $\sqrt{2} = \frac{a}{b}$ such that a, b have no common factors. ($\frac{3}{4}$ not e.g. $\frac{6}{8}$. as 2 is a factor of 6 and also of 8.)
- Then $2 = (\frac{a}{b})^2 = \frac{a^2}{b^2}$, so $a^2 = 2b^2$.
- Thus a is even.
- So $a = 2c$. Thus $2b^2 = (2c)^2 = 4c^2$.
- So $b^2 = 2c^2$. Thus b is also even.
- This is a contradiction.
- Why is this one beautiful? It involves deep new ideas, but nevertheless is simple, and changes the course of history.
- Again, I think it is the cognitive stimulation. The problem solving, like humour, but that's another story.

A beautiful formation

- Immodestly, this is a formation I devised and is called the Rose.
- Why? The visualization in space.
- The phrasing. The feel of the formation. The flow.
- I used `https://www.youtube.com/watch?v=5CxvlnDB74U&t=116s`

A beautiful dance

- Here's one using the rose progression.
- The Library at Birmingham
- I used `https://www.youtube.com/watch?v=1OkhedA9rIA&feature=emb_logo`

Summary

- So for this kind of mathematics we attempt to understand interacting processes as they change with time, and develop algorithms to cope with that.
- The creative part is to develop the framework: How to represent things, and the algorithmics: How to make efficient, safe, etc algorithms, and how do I know they are good or maybe the best?
- For this kind of dancing we have many agents (often called people) interacting with time: How to develop an algorithm (often called a dance), choreography which allows flow through positions.
- Both have aesthetics but at least the mathematics is not usually set to music.

- You can live you life to music....
- I used `https://www.youtube.com/watch?v=i870lQRyPJI` and began at 1 minute.

Thank You