On Roman Emperors, their Chinese Counterparts and Statistics of their Reign

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1 Introduction and main observation

Fitting a statistical model to data is always a task of interest and importance. It is useful to have empirical curves smoothed and histograms approximated. But it becomes extremely interesting and important when this fitting gives you an insight into the nature of the random phenomena behind the data. This happens when we fit one of the distributions which have some clearly understood random mechanism behind them. Perhaps one of the most vivid distributions of this kind is the exponential distribution (see eqn (1) below).

Statisticians know very well that if in a course of time some events, like emissions of radioactive particles, traffic accidents, occurrence of abnormally high prices on stable markets, etc., take place at random moments and if the random times elapsing between them have exponential distribution then these events occur purely at random, in an unexpected, unprepared and unpredictable way.

The life-lengths of human beings can not have the nature of exponentially distributed random variables – human organisms accumulate damages, suffer from illnesses and stresses and, in a word, are aging. Apart from pure accidents, humans die as the result of this aging.

Fig. 1 left shows the graph of the empirical distribution function of New Zealand males and females (Statistics NZ, 2004). There is not the slightest resemblance to the form of an exponential distribution, which is concave function for any $\lambda > 0$. The graph on the right shows the empirical estimation of the failure rate or force of mortality $\alpha(x) = f(x)/[1-F(x)]$ with $f(x) = dF(x)/dx$ for New Zealand males and females. Again not the slightest resemblance to the constant failure rate $\alpha(x) = \lambda$ of an exponential distribution.

It seemed to us that the same must be the case with rulers, kings, governors, and such like: they must have stopped ruling, reigning and governing as a consequence of accumulated controversies, tensions and damages of economic, social, political or personal nature. Contrary to this, looking at the data on the lengths of rule of Western Roman Emperors of the period of decline and fall (taken, conditionally, from Nerva (96-98) to Theodosius I (379-375)), which we reproduce below, we found agreement with the exponential distribution, which seems remarkably good for such a complex and long spanned historical data.

Hence we are led to the conclusion that Roman Emperors had stopped ruling (and, in many cases, died) not as a certain consequence of accumulated controversies or tensions,

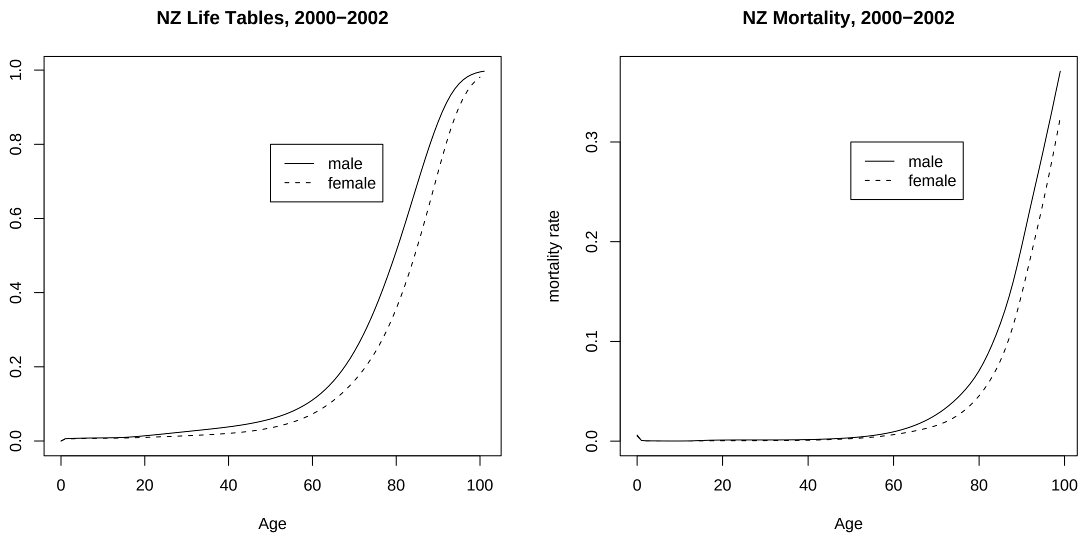

Figure 1: Life distribution for New Zealand males and females (left) and the rates of mortality, 2000-2002. Data from Statistics NZ.

but did this "purely at random", in unexpected, unprepared and unpredictable ways.

Our first impression was that this remarkable instability of supreme rule is a statistical manifestation of the complex phenomena which historians call "decline and fall". However, our second example suggests that this impression was incorrect: the pattern of exponentiality can be found in a much wider class of the historic periods than those which can be labeled as "decline and fall" periods.

Indeed, in search of an alternative example of non-exponential reign lengths we considered data for Chinese Emperors (from 770 BC to 1644 AD) throughout the colossal span of their enormous history. In contrast to our prior expectations the data again showed surprisingly good agreement with the pattern of exponential distribution.

This can be observed more often than one could expect. The further example can be found in rule lengths of European monarchs of new times (starting with, say, the 16th century). We illustrate how good this agreement can be in §3 where we present some goodness of fit results. It is, however, possible, especially in the case of monarchs of later times, that we observe only a similarity in the pattern rather than "genuine" exponentiality of the distribution. In §4 we consider two competing explanations for the exponential pattern and, as we hope, later, in §5 we demonstrate both possibilities by historical data. For Roman Emperors we believe we indeed have exponentiality of their rule lengths with all the interpretation it invokes.

2 How did we test exponentiality

To test the hypothesis that a sample $T_1, T_2, \dots, T_n$ of n independent random variables follows an exponential distribution

$$F(x,\lambda) = 1 - e^{-\lambda x} \tag{1}$$

with some apriori unspecified $\lambda > 0$ we used the approach suggested in (Khmaladze, 1981). Application of this approach to testing exponentiality was recently considered in detail in (Haywood and Khmaladze, 2005). Namely one could consider an empirical distribution function $F_n(t)$ of this sample and compare it to $F(t,\hat\lambda)$ with λ estimated by $\hat\lambda$ from this same data. There is, however, some advantage in considering not the difference $F_n(t) - F(t,\hat\lambda)$ but $F_n(t) - K(t,F_n)$ where $K(t,F_n)$ is called the "compensator" in the martingale theory of point processes (see, e.g., Liptser and Shiryaev, 2001, vol.2), and can be thought about as a somewhat more flexible modification of $F(t,\hat\lambda)$. The form of $K(t,F_n)$ for the family of exponential distributions is particularly simple:

$$\begin{aligned} K(t,F_n) &= \hat\lambda \int_0^\infty \left[2 + \frac{\hat\lambda}{2}\min(t,\tau) - \hat\lambda\tau\right] \min(t,\tau) F_n(d\tau) \\ &= \frac{\hat\lambda}{n}\sum_{i=1}^n \left[2\min(t,T_i) + \frac{\hat\lambda}{2}\min{}^2(t,T_i) - \hat\lambda T_i \min(t,T_i)\right] \end{aligned} \tag{2}$$

or

$$K(t,F_n) = \frac{\hat\lambda}{n}\sum_{i:T_i\le t}\left[2T_i - \frac{\hat\lambda}{2}T_i^2\right] + \hat\lambda\left[2t + \frac{\hat\lambda}{2}t^2\right](1 - F_n(t)) - t\frac{\hat\lambda^2}{n}\sum_{i:T_i>t} T_i\,.$$

To see that $K(t,F_n)$ is really "certain modification" of $F(t,\hat\lambda)$ one can check that $K(t,F(t,\lambda)) = F(t,\lambda)$.

If the hypothesis of exponentiality is true, the version of the empirical process $w_n(s), s \in [0,1]$, defined as

$$w_n(s) = \sqrt{n}[F_n(t) - K(t,F_n)], \quad s = F(t,\hat\lambda)$$

converges in distribution, as $n \to \infty$, to standard Brownian motion $w(s), s \in [0,1]$. Therefore for the distribution of the classical Kolmogorov-Smirnov statistic

$$D_n = \sqrt{n}\sup_{t\ge 0}|F_n(t) - K(t,F_n)| = \sup_{0\le s\le 1}|w_n(s)|$$

we obtain, for large n, the approximation

$$\mathbb{P}\{D_n \ge z\} \approx \mathbb{P}\{\sup_{0\le s\le 1}|w(s)| \ge z\} = G(z)$$

The explicit expression for $G(z)$ is

$$G(z) = \frac{4}{\pi}\sum_{n=0}^\infty \frac{(-1)^n}{2n+1}\exp\{-\frac{\pi^2(2n+1)}{8z^2}\}$$

and can be found, e.g., in (Shiryaev, 1999) or (Borodin and Salminen, 2002). Since w is a standard process the distribution $G(z)$ is also standard and depends neither on the form of $F(t,\lambda)$ nor on the value of λ. So, we have obtained what is called an asymptotically distribution-free testing procedure.

Convergence of w_n to w is not slow and $G(z)$ can be used as an approximation of $\mathbb{P}\{D_n \ge z\}$ for $n \ge 50$ with sufficient confidence (see Haywood and Khmaladze, 2005, §4). In particular $G(1.40) = 0.32$, $G(1.50) = 0.27$, $G(1.60) = 0.22$ while $G(1.95) = 0.10$. The tables of $G(z)$ can be found at `http://www.mcs.vuw.ac.nz/~ray/Brownian`.

3 Chronologies and goodness of fit results

We examine three versions of the chronological tables of Western Roman Emperors. The first one we use is the chronology given in (Kienast, 1990). The next was kindly made available to us by Prof. T. Parkin from University of Queensland. It contains also birth dates and hence the ages at ascent, which we use in the analysis below. Besides these two we use the chronology given in the classical work by Edward Gibbon, see (Gibbon, 1998). (The first volume of his history was published in 1776 and the whole "History" was completed in 1788. This is the chronology we started this study with several years ago.) Although these three chronologies are slightly different in certain details the main conclusion about the exponential pattern of the durations of rule lengths stays intact. In fact, the three empirical distribution functions, drawn from the three chronologies, are quite close to each other.

Chronology of Kienast (Kienast, 1990). We refer to it below as "K". In the list of emperors from Augustus Octavian to Theodossius there are $n = 64$ names. The chronology shows the date of ascent and abdication (or death) in many cases. In some cases, however, no specific day or month is suggested. In these cases we selected mid-points as described below for the Parkin data.

T. Parkin's chronology (Parkin,). We refer to this chronology as "P". In the list of emperors from Augustus Octavian to Theodossius there are $n = 70$ names. In the majority of cases it agrees with the chronology given in (Kienast, 1990), but there are a few differences. When several possible days or even several possible years are suggested, if the dates were not too far apart we selected mid-points or even 'average dates'. For example, dates "June, 251 – August, 253" for Trebonianus Gallus we interpreted as "15 June, 251 – 15 August, 253" when calculating duration of reign in days. Relative to the total of 729 days of his reign the possible error does not look large. Similarly, "early 244 – September or October 249" as reign dates for Philippus we interpreted as "30 January, 244 – 30 September, 249", which, again, does not seem to involve a big error relative to the total of 2070 days, etc. As historical statements concerning individual rulers these mid-points or averages would be an impractical solution, but as far as we consider the whole collection of rule lengths as a "statistical ensemble", replacement of two or more uncertain points by a mid-point will deform the pattern very little. If the possible dates differ essentially, like the dates of ascent of Constantius I, Galerius and Maximinus Daia, we asked Prof. Parkin for advice.

Chronology of E. Gibbon (Gibbon, 1998). We refer to it below as to "G". In the list of emperors from Augustus Octavian to Theodossius there are $n = 63$ names. This chronology specifies years of ascent and abdication and we calculated the length of reign in years in a quite straightforward way, as a difference. Some years in this chronology are different from the other two and even the list of emperors is different. For example, Gibbon does not include Clodius Albinus, Pescennius Niger or Quintillus in his list. Everybody interested in later Roman history knows that it is not always straightforward to decide which person should be recognised as an Emperor (Augustus or Caesar) and from what date. There were many contenders to the purple, and many were proclaimed as emperors but whether they actually reigned or not is another question. The final judgment is, and should remain, with the historian.

In all three chronologies the reader will notice the simultaneous or parallel rule of

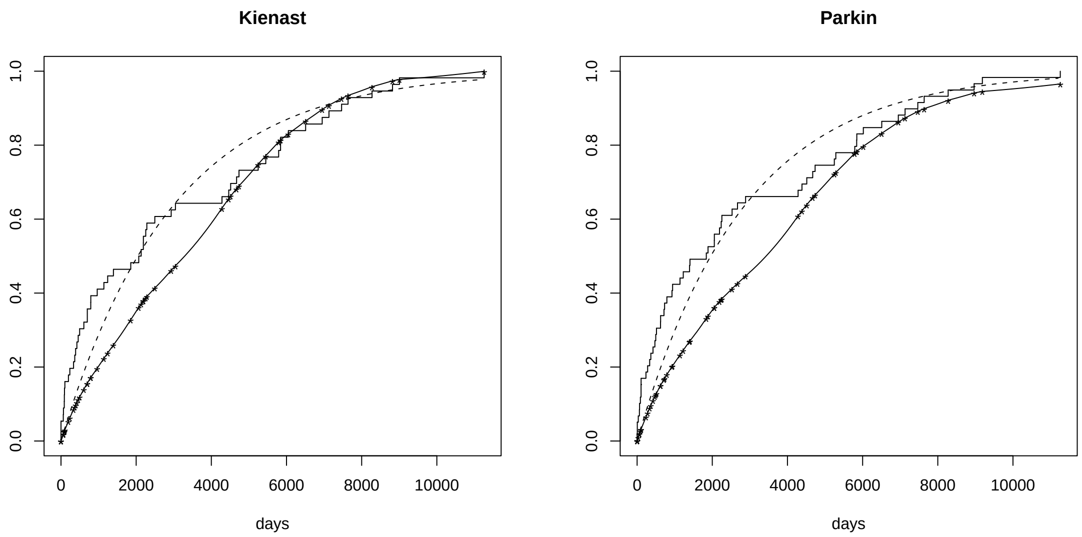

Figure 2: Empirical distribution function of durations of rule of Roman Emperors from "K" and "P" with corresponding exponential approximations (dashed lines). The lines with asterisks show the graphs of the compensators $K(t, F_n)$.

two or sometimes more Emperors. This situation was not uncommon in the vast Roman Empire.

In Fig. 2 left we show the empirical distribution function of the rule lengths calculated from "K" along with the approximating exponential distribution function and the graph of the compensator (2). The value of Kolmogorov-Smirnov statistic in this case is $D_n = 0.208\sqrt{53} = 1.51$ which corresponds to P-value of 0.26. Therefore the hypothesis of exponentiality can be easily accepted.

The situation with the data from "P" is very similar; in Fig. 2 right the value of the Kolmogorov-Smirnov statistic is $D_n = 0.225\sqrt{59} = 1.73$ with P-value 0.17.

Agreement between data from "G" and the the exponential distribution is also good as can be seen from Fig 3. The value of the Kolmogorov-Smirnov statistic is $D_n = 0.228\sqrt{52} = 1.65$ which corresponds to a P-value of 0.20.

We use a *chronology of Chinese emperors* as it is given in (Encyclopedia Britannica, 2002, "China", vol.3, p.222). In Fig. 4 we started with Tung (Eastern) Chou dynasty, the first Emperor in the list being Chi I-chia (P'ing-wang) (770-719 BC) and ended with Chu Yu-chien (1627-1644 AD), the last Emperor of the Ming dynasty. The total number of names in the list we analysed is therefore $n = 367$.

Agreement between the data from this chronological list and the exponential distribution function looks very good. However, curiously enough, the Kolmogorov-Smirnov test rejects exponentiality: the value of the statistic is $D_n = 0.128\sqrt{367} = 2.46$ which corresponds to P-value of 0.03. From other considerations we know that exponentiality here should be rejected, but we will discuss this in more detail in another publication.

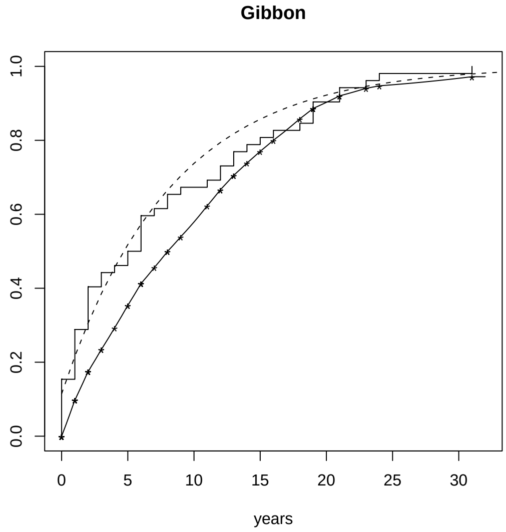

Figure 3: Empirical distribution function of durations of rule of Roman Emperors from "G" with corresponding exponential approximation (dashed line) and compensator.

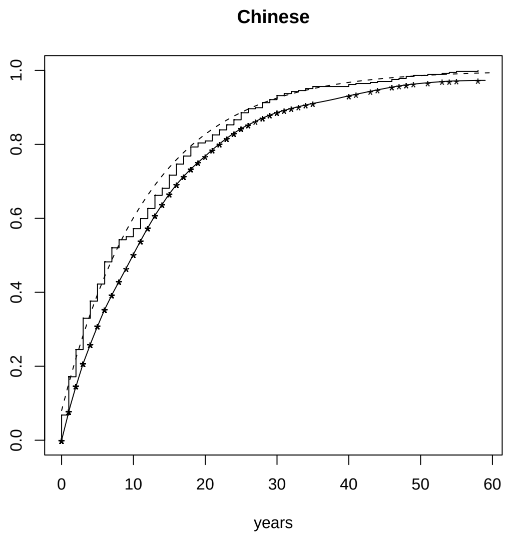

Figure 4: The empirical distribution function of durations of rule of Chinese Emperors along with exponential approximation (dashed line). The line with asterisks gives the graph of the compensator $K(x, P_n)$.

4 Explanations for exponentiality we consider.

In the case of rulers like Roman Emperors or Chinese Emperors of old times one can, perhaps, argue that their lives were constantly exposed to turmoil and life threatening challenges, no matter whether in peace or at war, and therefore there we can expect phenomena similar to deaths from accidents, and hence, exponential distributionness of the durations of reign. This sort of reasoning does not seem to us sufficient. Why should these life threatening challenges, plots and uprisings occur completely spontaneously and not as the result of accumulation of controversies and tensions? Or, to put it differently, if there are many challenges and threats to a reign, there usually are also strong means to protect the reign and the ruler. Therefore only exceptionally strong challenges could stop the reign and overthrow the ruler. In probability theory we know that typically the occurrences of an unusually high level of a stationary random process form a Poisson process, see, e.g., (Cramér and Leadbetter, 1967) or (Resnik, 1987) with, therefore, exponential distribution of interoccurence times.

There is, however, another, much simpler and trivial possible explanation for the exponential pattern we observe, which is the following. Before acquiring the title and position of Emperor the person typically should have had long political or military career and therefore must have been a person of mature age. The distribution function of the remaining life-time of a person of age X is the rescaled tail distribution

$$F(t|X) = \frac{F(X+t) - F(X)}{1 - F(X)}$$

of the distribution function F of life-times in the general population. This tail distribution, as one can see, for example, from Fig. 5, is indeed a concave function of t for reasonably large X, thus resembling an exponential distribution. The mixture of distribution functions $F(\cdot|X)$ over different ages of ascent may also be a distribution function similar in shape to an exponential distribution function and it is only this similarity that we are observing.

According to the first hypothesis, the nature and statistical properties of the remaining life of an Emperor was changing after ascent in a very clear way – the remaining life became more haphazard and vulnerable. According to the second hypothesis, the fact of ascent did not affect the remaining life distribution of the person, or affected it very little.

To obtain the first numerical impression of the problem, we considered the empirical distribution function of the life-times of, say, New Zealand males which show the median age at death equal 80 years. We scaled it down to make the median age at death smaller – firstly equal 60 years, then 50 years then 40 years ($c = 1.33$, $c = 1.60$ and $c = 2.0$). Then, using the ages $X_i, i = 1, \dots, n$, at ascent given in "P" we calculated the mixed remaining life distribution

$$H(t) = \frac{1}{n} \sum_{i=1}^{n} F(t|X_i)$$

of the remaining life distributions. In both cases it indeed is a concave function of t, but still different from an exponential distribution. The graph of $H(\cdot)$ with median age at death of 50 years, which was the closest fit to an exponential distribution, is given in Fig. 5 left.

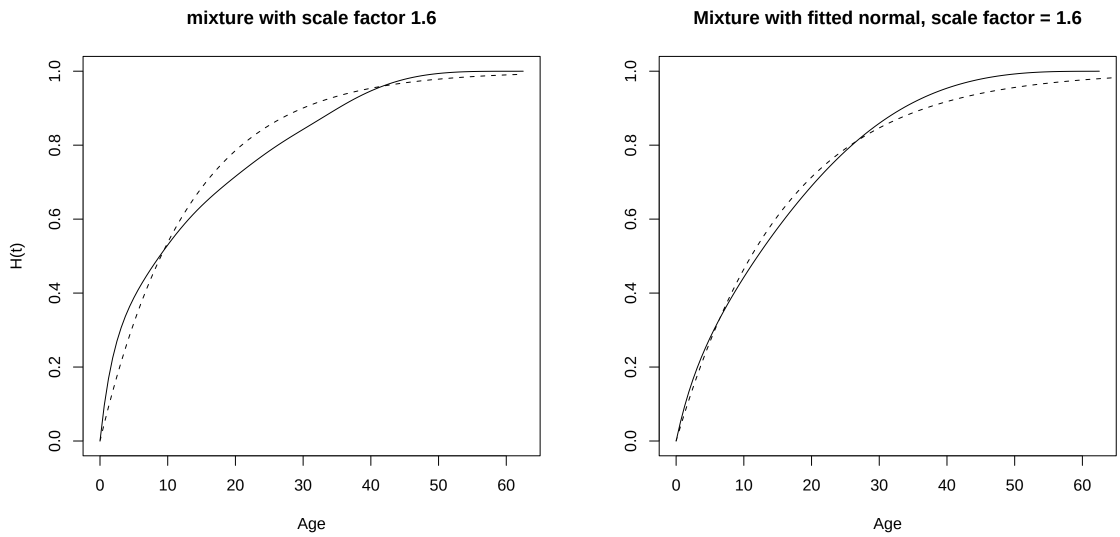

Figure 5: Mixtures of the remaining life distribution of NZ males (a) using ages at ascent from "P" (left) and (b) using normal mixing distribution. The life distribution is scaled down to median age at death of 50 years. The parameters of mixing normal distribution were $\mu = 40$ years and $\sigma = 18$ years. In both cases, the approximating exponential is shown as a dashed line.

Instead of using empirical values for mixing ages one could use, say, a fitted normal distribution, truncated at ages 0 and 80, and consider

$$\tilde{H}(t) = \int_0^{80} F(t|x)\Phi_{(\mu,\sigma)}(dx)dx/(\Phi_{(\mu,\sigma)}(80) - \Phi_{(\mu,\sigma)}(0))$$

where $\Phi_{(\mu,\sigma)}(x)$ denotes the normal distribution function. The parameters of the fitted normal distribution were $\mu = 40$ years and $\sigma = 18$ years. Fig. 5 right shows that $\tilde{H}$, again, resembles an exponential distribution function although not better than H.

Therefore, to distinguish between an exponential distribution and the mixed remaining life distribution can be a difficult statistical problem when the number of observations, as in the present case, is not large.

5 Further statistical analysis and comparison with some European monarchies.

The first natural step could be to look at the rate of mortality (failure rate). In Fig. 6 we show the (empirical) rate of mortality $\hat{\alpha}$ corresponding to the data on Roman Emperors from "K". It looks to us quite as a random function which fluctuates around a constant level – just as it has to be in the case of an exponential distribution. However, conclusive analysis based on the rate of mortality requires actually even larger sample sizes (for the theory of this analysis see, e.g., Andersen et al, 1993).

Figure 6: Empirical mortality of Roman Emperors

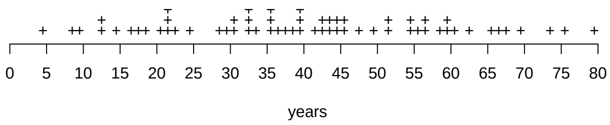

Figure 7: Timeline of age of ascent of Roman Emperors

In the data from “P” one can see that ages of ascent indeed had a very wide range: the data more or less agrees with the uniform distribution of the age at ascent from 0 to, say, 75 years and, as we said, agrees well with truncated normal distribution with expected value 40 years and very large standard deviation of 18 years (see Fig. 7). In other words it is incorrect to think that emperors were typically persons of mature age; many of them were middle aged or young and some of them were very young.

One can ask now whether Emperors who started the reign at quite a young age, did really live as long as could be expected from a “common” member of population of the same age. Fig. 8 shows age of ascent and the duration of reign for each Emperor. (Here we excluded Diocletian, Constantius I, Galerius, Maxentius, Maximinus Daia because of a large uncertainty in their possible ages at ascent.) Along with the data points we show by the dotted line the average remaining life at each age as it is known from Ulpian’s Tables. Visually it is quite clear that younger Emperors lived much shorter lives than what the Tables suggest – not a single data point for ages at ascent below 33 years exceeds the average level.

Historians, and especially those who study historical demography, may differ in opinion whether Ulpian’s Tables should be taken as completely true and correct averages of remaining lives (see, e.g., Kopf, 1927). However, whatever the point of view on this

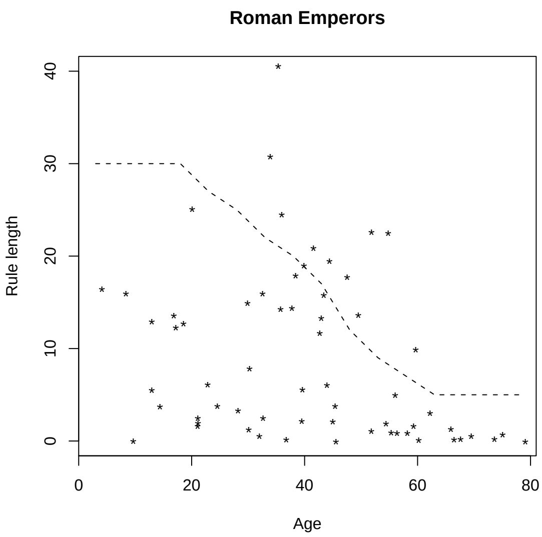

Figure 8: Rule length of Roman Emperors by age

subject, one can proceed without any reference to these tables. Indeed, since based solely on the durations of rule it is difficult to distinguish between two explanations because they lead to distributions close to each other, we can use one more variable, the age at ascent, in our analysis. If the rule lengths are closely related to the remaining lives, then the younger the age of ascent the longer the person should rule (and live). If, however, we have the "genuine" exponentiality of rule lengths, then, for each ruler, his age of ascent X_i and the duration of his reign T_i should be independent and the latter should be exponential.

To use a relevant statistical test we first excluded the group of 9 Emperors whose ages at ascent were over 60 years. Indeed, for this group of persons the hardship of being Emperor was combined with relatively old age, and it is only very natural that they did not rule long. The remaining data we divided into two groups of equal size: those who ascended at the age ≤ 36 or > 36 and Fig. 9 shows their empirical distribution functions. They look very convincingly as coming from the same distribution, and there is no need for any formal statistical test. Nevertheless, note that the value of two sample Kolmogorov-Smirnov statistic is 0.612 which is below its median value.

It would be nice to have an example of rule lengths which are not exponential. To find such example we considered the data of some of European monarchies of later times (from the 15th century to the early 20 century). One could expect relatively stable reigns and hence the reign lengths not following an exponential distribution.

Indeed, although the data on durations of reign of $n = 23$ Spanish kings, (starting with Isabella I, reference `http://en.wikipedia.org/wiki/List_of_Spanish_monarchs`), does not reject the hypothesis of exponentiality, with the value of the statistic $D_n = 0.395\sqrt{23} = 1.90$ corresponding to a P-value of 0.12, independence between X_i and T_i is fully rejected, as we show in Fig. 11. Note that this happens in spite of the fact that in a number of cases the end of reign was indeed an abdication and not the end of life and that in several cases the same person ruled several times but we counted these periods

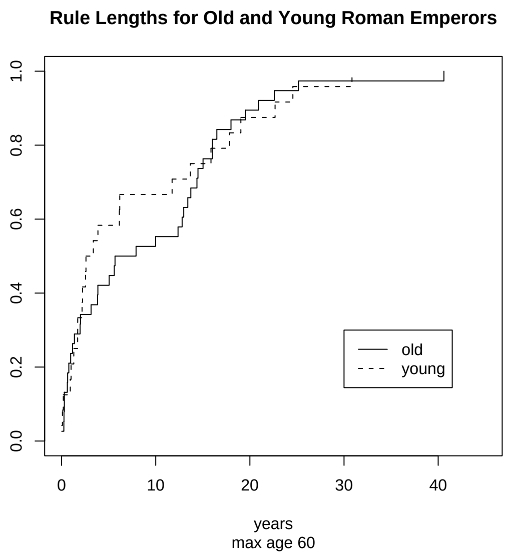

Figure 9: Rule lengths for old and young Roman Emperors

separately.

The data for Romanov dynasty of Russia agrees with exponentiality much more closely, with the value of the statistic $D_n = 0.244\sqrt{24} = 1.20$ corresponding to a p-value of 0.54. At the same time the data supports the hypothesis of independence between X_i and T_i (see Fig. 13 left). One should note that there was some turmoil on the throne and forced abdications in the history of this dynasty. The agreement with exponentiality would be even closer if we included data starting from the time of Ivan the Terrible.

Another vivid example of stability emerges from the data on British monarchs. Although the collection of their rule lengths, starting with Henry VII and finishing with George VI (data from Brittania web site http://www.britannia.com/history/h6f.html) as a whole agrees with exponentiality quite well, with the value of the statistic $D_n = 0.219\sqrt{22} = 1.03$ corresponding to a p-value of 0.40, the scatter plot shows very clearly that younger monarchs indeed ruled longer. Although the number of observations here is only 22 visually the picture is very clear.

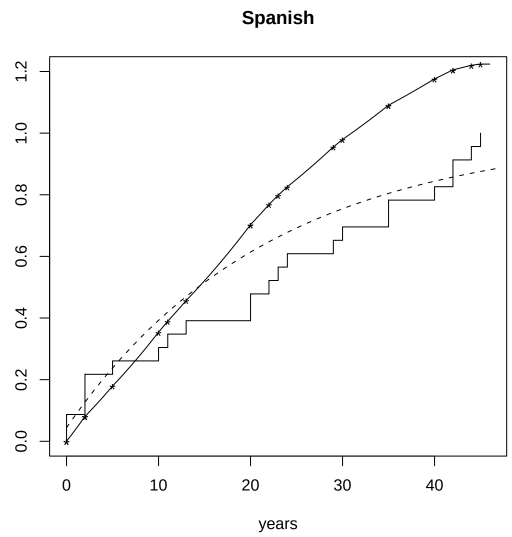

Figure 10: Empirical distribution function of durations of rule of Spanish Rulers with corresponding exponential approximation (dashed line) and compensator.

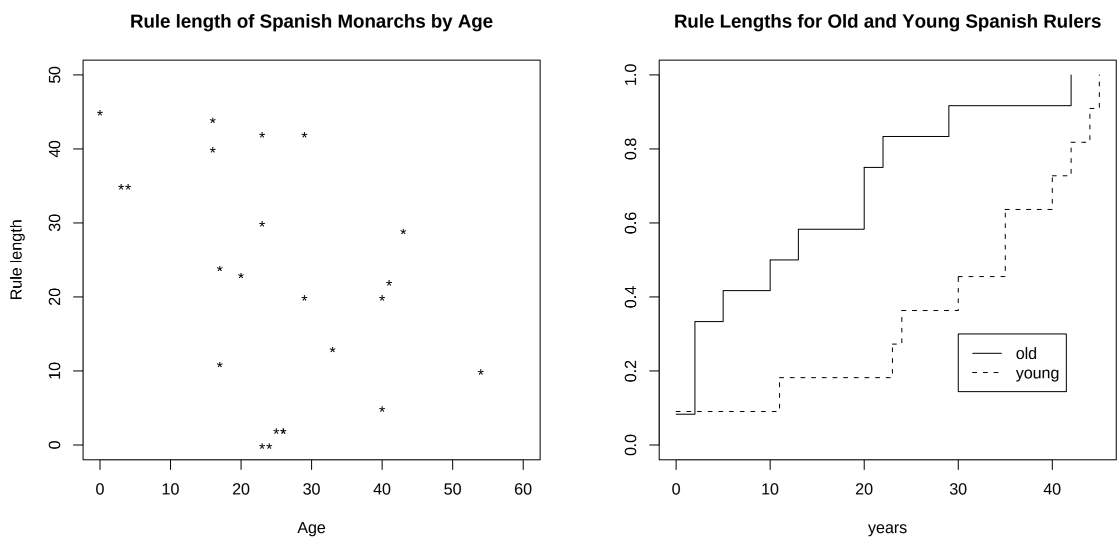

Figure 11: Rule length of Spanish Rulers by age (left) and comparison of young with old (right)

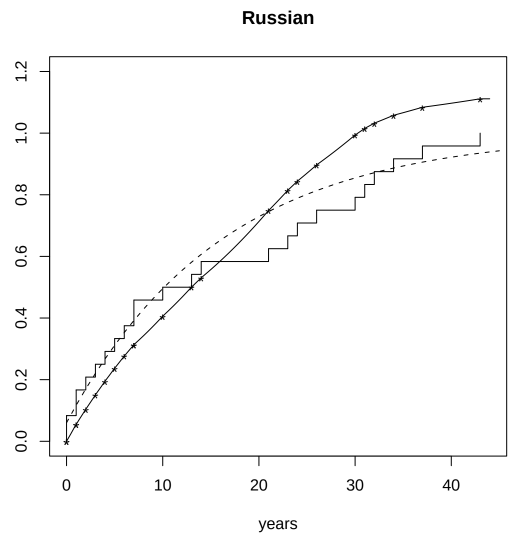

Figure 12: Empirical distribution function of durations of rule of Russian Rulers with corresponding exponential approximation (dashed line) and compensator.

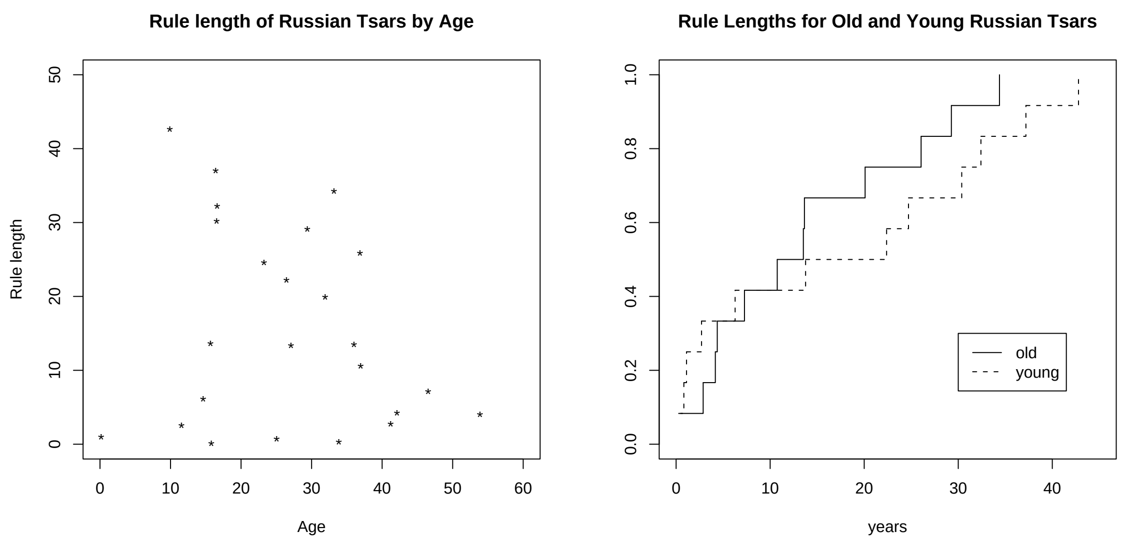

Figure 13: Rule length of Russian Rulers by age (left) and comparison of young with old (right)

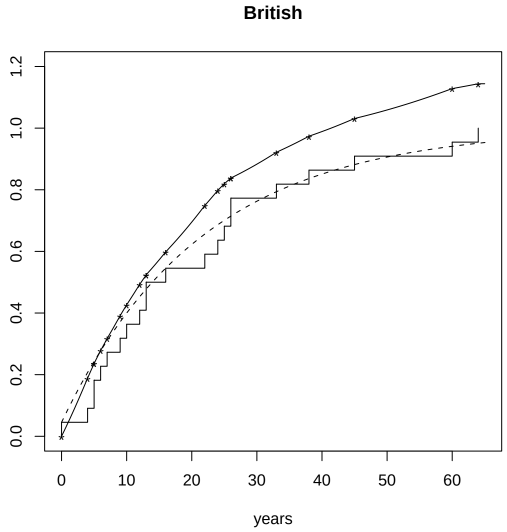

Figure 14: Empirical distribution function of durations of rule of British Rulers with corresponding exponential approximation (dashed line) and compensator.

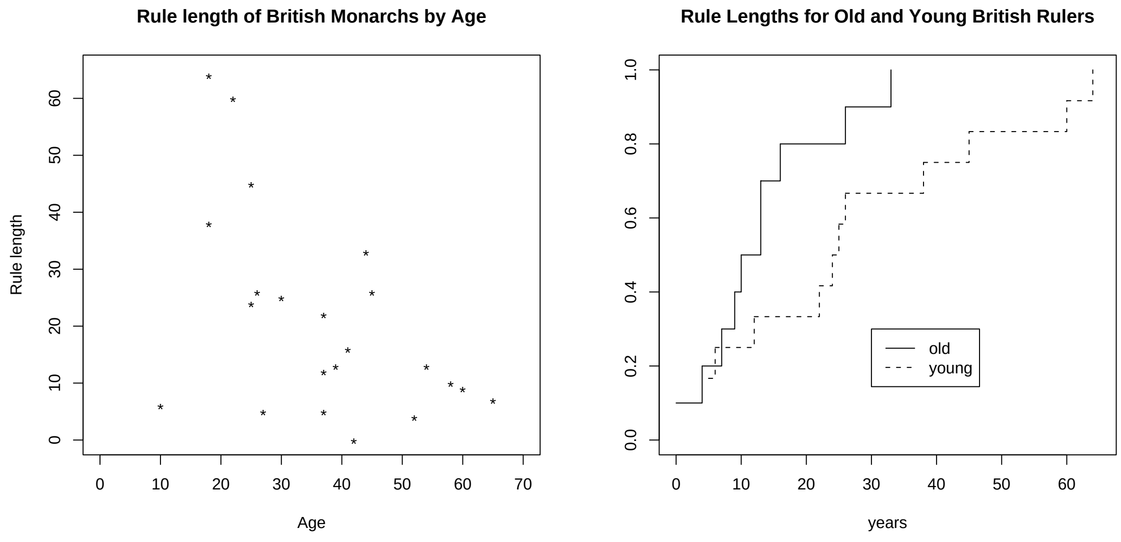

Figure 15: Rule length of British Rulers by age (left) and comparison of young with old (right)

A Gibbon Chronology

B.C. A.D.	
27-14	Augustus
14-37	Tiberius
37-41	Gaius (Caligula)
41-54	Claudius
54-68	Nero
68-69	Galba
69	Otho
69	Vitellius
69-79	Vespasian
79-81	Titus
81-96	Domitian
96-98	Nerva
98-117	Trajan
117-138	Hadrian
138-161	Antoninus Pius
161-180	Marcus Aurelius with Lucius Verus 161-169 and Commodus from 177
180-192	Commodus
193	Pertinax followed by Didius Julianus
193-211	Septimius Severus with Caracalla from 198 and Geta from 209
211-217	Antonius (Caracella) with Geta (211-212)
217-218	Macrinus with Diadumenianus in 218
218-222	Antonius (Elagabalus)
222-235	Severus Alexander
235-238	Maximinus Thrax
238	Gordian I, Gordian II, Pupienus (Maximus) and Balbinus
238-244	Gordian III
244-249	Philip the Arab with his son Philip 247-249
249-251	Decius
251-253	Trebonianus Gallus and Volusianus
253-260	Valerian with Gallienus
260-268	Gallienus
268-270	Claudius II Gothicus
270-275	Aurelian
275-276	Tacitus (and Florianus 276)
276-282	Probus
282-283	Carus
283-284	Carinus and Numerian
284-305	Diocletian }
286-305	Maximian } Both abdicated in 305
305-311	Galerius, associated with him over various periods Constantius I Chlorus, Severus II, Licinius, Constantine I, and Maximinus Daza. In 309 there were six Augusti
311-324	Constantine I and Licinius
324-337	Constantine I
337-340	Constantine II, Constantius II and Constans
340-350	Constantius II and Constans

350-361	Constantius II
361-363	Julian
363-364	Jovian
364-375	Valentinian I and Valens with Gratian from 367
375-378	Valens, Gratian and Valentinian II
378-395	Theodosius I (the Great) reigned with Gratian and Valentinian II from 378-383, with Valentinian II and Arcadius 383 to 392, and with Arcadius and Honorius 392 until his death in 395

B Kienast Chronology

Augustus (16. Jan. 27 v. Chr.-19. Aug. 14 n. Chr.)
Tiberius (19. Aug. 14-16. Marz 37)
Caligula (18. Marz 37-24. Jan. 41)
Claudius (24. Jan. 41-13. Okt. 54)
Nero (13. Okt. 54-9. Juni 68)
Galba (8. Juni 68-15. Jan. 69)
Otho (15. Jan.-16. April 69)
Vitellius (2. Jan.-20 Dez. 69)
Vespasian (1. Juli 69-23 Juni 79)
Titus (24. Juni 79-13. Sept. 81)
Domitian (14. Sept. 81-18. Sept. 96)
Nerva (18. Sept. 96-27.[?] Jan. 98)
Trajan (28. Jan. 98-7. Aug. 117)
Hadrian (11. Aug. 117-10. Juli 138)
Antoninus Pius (10. Juli 138-7. Marz 161)
Mark Aurel (7. Marz 161-17. Marz 180)
Commodus (17. Marz 180-31. Dez. 192)
Pertinax (31. Dez. 192-28. Marz 193)
Didius Iulianus (28. Marz-1. Juni 193)
Septimius Severus (9. April 193-4. Febr. 211)
Caracalla (4. Febr. 211-8. April 217)
Macrinus (11. April 217-8. Juni 218)
Elagabal (16. Mai 218-11. Marz 222)
Severus Alexander (13. Marz 222-Febr./Marz 235)
Maximinus Thrax (Febr./Marz 235-Mitte April [?] 238)
Gordian I. (Jan [?] 238)
Gordian II. (Jan [?] 238)
Pupienius (Ende Jan./Anf. Febr. [?]-Anf. Mai [?] 238)
Balbinus (Jan./Febr. [?]-Mai [?] 238)
Gordian III. (Jan./Febr. [?] 238-Anf. 244)
Philippus Arabs (Anf. 244-Sept./Okt. 249)
Decius (Sept./Okt. 249-Juni 251)
Trebonianus Gallus (Juni [?] 251-Aug. [?] 253)
Aemilius Aemilianus (Juli/Aug.-Sept./Okt. 253)
Valerian (Juni/Aug. 253-Juni [?] 260)
Gallienus (Sept./Okt. 253-ca. Sept. 268)
Claudius II. Gothicus (Sept./Okt. 268-Sept. 270)

Quintillus (September 270)
Aurelian (Sept. 270-Sept./Okt. 275)
Tacitus (Ende 275-Mitte 276)
Florianus (Mitte-Herbst 276)
Probus (Sommer 276-Herbst 282)
Carus (Aug./Sept. 282-Juli/Aug. 283)
Numerianus (Juli/Aug. [?] 283-Nov. 284)
Carinus (Fruhjahr 283-Aug./Sept. 285)
Diocletian (20. Nov. 284-1. Mai 305)
Maximian (Okt./Dez. 285-ca. Juli 310)
Constantius I. (1. Marz 293-15. Juli 306)
Galerius (21 Mai [?] 293-Anf. MAi 311)
Maximinus Daia (1. Mai 305-Spatsommer 313)
Severus II. (1. Mai 305-Marz/April 307)
Maxentius (28. Okt. 306-28. Okt. 312)
Licinius (11. Nov 307-19 Sept. 324)
Constantin I. (25. Juli 306-22. Mai 337)
Constantin II. (9. Sept. 337-Anf. April 340)
Constans (9. Sept. 337-18. Jan. 350)
Constantius II. (9. Sept. 337-3. Nov. 361)
Julian (ca. Febr. 360-26./27. Juni 363)
Jovian (27. Juni 363-17. Febr. 364)
Valentinian I. (25. Febr. 364-17. Nov. 375)
Valens (28. Marz 364-9. Aug. 378)
Gratian (24. Aug. 367-25. Aug. 383)
Valentinian II. (22. Nov. 375-15. Mai 392)
Theodosius I. (19. Jan. 379-17. Jan. 395)

C Parkin Chronology

Augustus
23 Sept. 63 BC
16 Jan. 27 BC, aged 35
19 Aug. AD 14, aged 75 yrs

Tiberius
16 Nov. 42 BC
19 Aug. AD 14, aged 54
16 (26 Dio?) March 37, aged 77

Caius
31 August AD 12
18 (26 Dio?) March 37, aged 24
24 (22?) Jan. 41, aged 28

Claudius
1 August 10 BC

24 Jan. AD 41, aged 49
13 Oct. 54, aged 63

Nero
15 Dec. 37
13 Oct. 54, aged 16
9 June 68, aged 30

Galba
24 Dec 3 BC (probably, rather than 5 BC)
8 June 68, age 69
15 Jan 69, age 70

Otho
28 April 32
15 Jan 69, aged 36
16 April 69, aged 36 (few weeks short of 37)

Vitellius
7th or 24th of Sept. AD 12 or 15 (7/9/12 better)
2 Jan 69, aged 53 or 56
20/21 Dec 69, aged 54 or 57

Vespasian
9th or 14th or 17th Nov. AD 9
1 July 69, aged 59
23 June 79, aged 69 [some would say 24th June]

Titus
30 Dec 39
24 June 79, aged 39
13 Sept. 81, aged 41

Domitian
24 Oct 51
14 Sept 81, aged 29
18 Sept 96, aged 44

Nerva
8 Nov. 30 (or 35 ?)
18 Sept 96, aged 60 or 65
27/28 Jan. 98, aged 62 or 67

Trajan
18 Sept. 53 (56?)
28 Jan 98, aged 44 (41?)
7/8 (11?) Aug. 117, aged 63 (60?)

Hadrian

24 Jan 76
11 Aug 117, aged 41
10 July 138, aged 62

Antoninus Pius
19 Sept 86
10 July 138, aged 51
7 March 161, aged 74 years 5 months 18 days

Marcus Aurelius
26 April 121
7 March 161, aged 39
17 March 180, aged 58 years, 11 months

Lucius Verus
15 Dec 130
7 March 161, aged 30
Jan or Feb 169, aged 38

Commodus
31 Aug 161
late 176 / early 177, aged 15. OR 17 March 180 (death of Aurelius), aged 18
31 Dec. 192, aged 31

Pertinax
1 August 126
31 Dec 192, aged 66
28 March 193, aged 66

Didius Iulianus
30 Jan 133
28 March 193, aged 60
1 June 193, aged 60

Septimius Severus
11 April 145 or 146
9 April 193, aged 46 or 47 (2 days short of birthday)
4 Feb 211 aged 64 or 65

Clodius Albinus
25 November 147 (thus SHA - fiction!)
April 193, aged 45?
19 Feb 197, aged 49?

C. Pescennius Niger (Iustus)
AD 135-140 ?
April 193, aged 53-58?
April 194, aged 54-59?

Caracalla
4 April 188 (186 ?)
28 Jan or 8/9 April 198, aged 9/10 (11/12 ?) OR 4 Feb 211, aged 22 (24 ?)
8 April 217, aged 29 (31 ?)

Geta
7 March 189
Sept or Oct 209 or 210, aged 20/21
Dec 211, aged 22

Macrinus
164 or 166
11 April 217, aged 52/53 or 50/51
8 June 218, aged 53/54 or 51/52

Diadumenianus
14 Sept 208
May 218, aged 9
June 218, aged 9

Elagabalus (Heliogabalus)
ca. 203/4
16 May 218, aged 13-15 (?)
11 March 222, aged 17-19 (?)

Severus Alexander
1 Oct 208 or 209
13 March 222, aged 12/13
Feb or March 235, aged 25/26

Maximinus Thrax
172/3
Feb or (mid-?)March 235, aged 61-63
April (early June?) 238, aged 64-66

Gordian I
158/9 (a little later) ? (ca. 178 ?)
Jan (mid/late March?) 238 ?, aged 78-80?
20 Jan (late April?) 238?, aged 78-80?

Gordian II
ca. 192
Jan (mid/late March?) 238, aged ca. 45
20 Jan (late April?) 238, aged ca. 45/46 ?

Balbinus
? (Kienast p.193: 'unbekannt und nicht zu ermitteln')
Jan or Feb (late April?) 238
May (early August?) 238

Pupienus
ca, 164 ?
Jan or Feb (late April?) 238, aged 74 (?)
May (early August?) 238, aged 74 (?)

Gordian III
20 Jan 225 or 226
238, aged 11-13 years
early 244, aged 18/19

Philippus
204 ??
early 244, aged 39/40 ??
Sept or Oct 249, aged 44/45 ??

Decius
190 or 200 ?
? Sept or Oct 249, aged 48/49 or 58/59 ?
June 251, aged 50/51 or 60/61 ?

Trebonianus Gallus
206?
June 251 ?, aged 44/45?
Aug 253 ?, aged 46/47?

Volusianus (son of Trebonianus Gallus)
ca. 230
August ? 251, aged 20/21
August ? 253, aged 22/23

Aemilianus
ca. 207 or 214 ?
July or Aug 253, aged 39/39 or 45/46 ?
Sept or Oct 253, aged 38/39 or 45/46 ?

Valerian
ca. 193 or 200 ?
ca, July 253, aged 53/60 ?
taken prisoner ca. June 260, aged 60/67 ?
after 262, aged 62+/69+ ?

Gallienus
ca. 213 or 218
Sept or Oct 253, aged 39/40 or 34/35
22 March, or in Sept., 268, aged 54/55 or 49/50

Claudius II Gothicus
10 May 214 ?

Sept or Oct 268, aged 54?
Sept 270, aged 56?

Quintillus (brother of Claudius II)
?
Sept 270
Sept 270

Aurelian
9 Sept 214 ?
Sept 270, aged 56?
Sept or Oct 275, aged 61?

Tacitus
ca. Sept 200 ?
late Sept 275, aged 74/75?
mid 276, aged 75/76

Florianus (brother of Tacitus)
?
mid 276
autumn 276

Probus
19 Sept 232
summer/autumn 276, aged 43/44
autumn 282, aged 49/50

Carus
ca. 224 ?
Aug or Sept 282, aged 58?
July or Aug 283. aged 59?

Carinus (older [?] son of Carus)
ca. 250
early 283, aged 32/33?
Aug or Sept 285, aged 34/35?

Numerianus (younger [?] son of Carus)
ca. 253
July or Aug 283, aged 29/30?
Nov 284, aged 30/31?

Diocletian
22 Dec., 236/7 or 244 or 245 or 247/8 ?? (or even as early as ca. 225 ?!)
17 or 20 Nov 284, aged 35-47 (if 245, then aged 38)
abdicated 1 May 305, aged 56-68 (if 245, then aged 59)
3 Dec 311 or 312 or 313 or 315 or 316, aged 63-79 (if 245, then aged 67 in 313)

Maximian
21 July (nicht 22 Dec, says Kleinast) 249/250
Oct or Dec 285, or 1 April 286, aged 35/36
abdicated 1 May 305, aged 54/555
back again late 306, aged 56/57
ca. July 310, aged 60/61

Constantius I
31 March ca. 250
1 March 293, or 1 May 305, aged ca. 43/55
25 July 306, aged ca. 56

Galerius
250s
21 May 293 or 1 May 305, aged 33-43 or 45-55 ?
May 311, aged 51-61 ?

Severus II
?
1 May 305 or 306
March or April 307

Maxentius
ca. 275-287
28 Oct 306, aged 18-31
28 Oct 312, aged 24-37

Constantine
27 Feb 272 or 273 (ca. 280, Nixon)
25 July 306, aged 33/34
22 May 337, aged 64/65

Licinius
ca. 265
11 Nov 308, aged ca. 43
resigned 19 Sept 324, aged ca. 59
spring 325, aged ca. 60

Maximinus Daia
20 Nov ca. 270 or 285
1 May 305 or 308 or 309 or 310, aged 19/34 or 22/37 or 23/38 or 24/39
mid 313, aged 27/42

Constantine II
7 August 316 OR Feb 217 ?
9 Sept 337, aged 20/21
early April 340, aged 23

Constans I

320 OR 323 ?
9 Sept 337, aged 14 or 17
ca. 18 Jan 350, aged 27 or 30

Constantius II
7 Aug 317
9 Sept 337, aged 20
3 Nov 362, aged 44

Julian
May or June, ca. 331 or 332
ca. Feb 360, aged 29?
26 or 27 June 363, aged 32?

Jovian
331
27 June 363, aged 32
17 Feb 364, aged 33

Valentinian I
321
25 Feb 364, aged 42/43
17 Nov 375, aged 54

Valens
ca. 328
28 March 364, aged 35?
9 Aug 378, aged 49?

Gratian
18 April 359
24 Aug 367, aged 8
25 Aug 383, aged 24

Valentinian II
autumn (not 2 July) 371
22 Nov 375, aged 4
15 May 392, aged 20

Theodosius I
11 Jan 346 or 347
19 Jan 379, aged 32/33
17 Jan 395, aged 48/49

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