Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui

Who's afraid of Lorentz symmetry breaking?

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Abstract:

Is Lorentz symmetry truly fundamental?
Or is it just an "accidental" low-momentum emergent symmetry?
Opinions on this issue have undergone a radical mutation over the last few years.
Historically, Lorentz symmetry was considered absolutely fundamental — not to be trifled with — but for a number of independent reasons the modern viewpoint is more nuanced.
What are the benefits of Lorentz symmetry breaking?
What can we do with it?
Why should we care?

Collaborators:

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Horava–Lifshitz gravity:

- Quantum Gravity at a Lifshitz Point.
  Petr Horava. Jan 2009.
  Phys.Rev.D79: 084008, 2009. arXiv: 0901.3775 [hep-th]
  (cited 120 times; 28 published).
- Membranes at Quantum Criticality.
  Petr Horava. Dec 2008.
  JHEP 0903:020, 2009. arXiv: 0812.4287 [hep-th]
  (cited 80 times; 18 published).
- Spectral Dimension of the Universe in Quantum Gravity at a Lifshitz Point.
  Petr Horava. Feb 2009.
  Phys.Rev.Lett.102: 161301, 2009. arXiv: 0902.3657 [hep-th]
  (cited 61 times; 12 published).

Horava–Lifshitz gravity:

As of 23 August 2009 Spires reports that this topic has generated:

- 3 published papers with 50 or more citations.
- 14 published papers with 25 or more citations.
- 21 published papers with 10 or more citations.

In addition Spires reports:

- 7 unpublished papers with 50 or more citations.
- 23 unpublished papers with 25 or more citations.
- 49 unpublished papers with 10 or more citations.

However you look at it, this topic is “highly active”.

Horava–Lifshitz gravity:

My interpretation of the central idea:

- Abandon ultra-high-energy Lorentz invariance as fundamental.
- One need "merely" attempt to recover an approximate low-energy Lorentz invariance.
- "Critical" Lifshitz point in $(d+1)$ dimensions $\Leftrightarrow$ Dispersion relation satisfies

$$\omega \to k^d \qquad \text{as} \qquad k \to \infty.$$

- To recover Lorentz invariance, at "low" momentum (but still including $k \gg m$) the dispersion relation should satisfy

$$\omega \to \sqrt{m^2 + k^2} \qquad \text{as} \qquad k \to 0.$$

QFT:

- Every QFT regulator known to mankind either breaks Lorentz invariance (e.g. lattice), or does something worse, something outright unphysical.
- For example:
  - Pauli–Villars violates unitarity;
  - Lorentz-invariant higher-derivatives violate unitarity;
  - dimensional regularization is at best a purely formal trick with no physical interpretation...
    (and which requires a Zen approach to gamma matrix algebra).

QFT:

Standard viewpoint:

- If the main goal is efficient computation in a corner of parameter space that we experimentally know to be Lorentz invariant to a high level of precision, then by all means, go ahead and develop a Lorentz-invariant perturbation theory with an unphysical regulator — hopefully the unphysical aspects of the computation can first be isolated, and then banished by renormalization.
- This is exactly what is done, (very efficiently and very effectively), in the "standard model of particle physics".

QFT:

Non-standard viewpoint:

- If however one has (even a very mild) reason to suspect that Lorentz invariance might ultimately break down at ultra-high energies, then a different strategy suggests itself.
- Maybe one could use the Lorentz symmetry breaking as part of the QFT regularization procedure?
- Could we at least keep intermediate parts of the QFT calculation "physical"?
- (Note that "physical" does not necessarily mean "realistic", it just means we are not violating fundamental tenets of quantum physics at intermediate stages of the calculation.)

QFT:

Consider a “physical” but Lorentz-violating regulator:

- Dispersion relation:

$$\omega^2 = m^2 + k^2 + \frac{k^4}{K_4^2} + \frac{k^6}{K_6^4}.$$

  We call this a “trans–Bogoliubov” dispersion relation.

- QFT propagator:

$$G(\omega, k) = \frac{1}{\omega^2 - \left[m^2 + k^2 + k^4/K_4^2 + k^6/K_6^4\right]}.$$

- In any (3+1) dimensional scalar QFT with arbitrary polynomial self-interaction this is enough (after normal ordering) to keep all Feynman diagrams finite.

QFT:

For details see:

- Lorentz symmetry breaking as a quantum field theory regulator.
  Matt Visser. Feb 2009.
  Phys.Rev.D80: 025011, 2009.
  arXiv: 0902.0590 [hep-th]
- Renormalization of Lorentz violating theories.
  Damiano Anselmi, Milenko Halat, Jul 2007.
  Phys.Rev.D76: 125011, 2007.
  arXiv: 0707.2480 [hep-th]
- Weighted scale invariant quantum field theories.
  Damiano Anselmi, Jan 2008.
  JHEP 0802:051, 2008.
  arXiv: 0801.1216 [hep-th]

QFT:

Lifshitz point of order z in $(d+1)$ dimensions:

- Dispersion relation:

$$\omega^2 = m^2 + k^2 + \sum_{n=2}^{z} g_n \, \frac{k^{2n}}{K^{2n-2}}$$

- QFT propagator:

$$G(\omega, k) = \frac{1}{\omega^2 - [m^2 + k^2 + \sum_{n=2}^{z} g_n \, k^{2n}/K^{2n-2}]}$$

- In a $(d+1)$ dimensional scalar QFT with $z = d$ and arbitrary polynomial self-interaction this is enough (after normal ordering) to keep all Feynman diagrams finite.

QFT:

For $d = z$ in $(d+1)$ dimensions, suppressing a proportionality constant :

- Dispersion relation:

$$\omega \to k^d \qquad \text{as} \qquad k \to \infty.$$

- QFT propagator:

$$G(\omega, k) \to \frac{1}{\omega^2 - k^{2d}} \qquad \text{as} \qquad k \to \infty.$$

- In the deep ultra-violet, rewrite

$$\omega \sim k^d \qquad \Leftrightarrow \qquad [\mathrm{d}t] \sim [\mathrm{d}x]^d,$$

which is the $z = d$ Lifshitz scaling condition.

Implications for quantum gravity:

Standard Einstein–Hilbert action:

$$\int \sqrt{-g_{3+1}} \; {}^{(3+1)}R \;\; \mathrm{d}^{3+1}x$$

Standard ADM decomposition:

$$\int \sqrt{-g_3} \;\; N \;\; \left\{ \mathrm{tr}[K^2] - \mathrm{tr}[K]^2 + {}^{(3)}R \right\} \;\; \mathrm{d}^3x \, \mathrm{d}t$$

Non-standard extension: Choose a "preferred foliation" with

$$\mathcal{L} = (\text{kinetic term}) - (\text{potential term})$$

Implications for quantum gravity:

Now generalize the potential
(beyond what you expect from Einstein–Hilbert).

Using the preferred foliation, perturb around flat 3-space:

$$\text{Riemann}(g_{ij} = \delta_{ij} + h_{ij}) \sim \sum_{n=0}^{\infty} h^n \, (\nabla^2 h + \nabla h \cdot \nabla h).$$

- So the spatial Riemann tensor leads to k^2 terms in the dispersion relation and propagator.

Horava–Lifshitz gravity:

- For $d = 3$, we want sufficient powers of k to improve Feynman diagram convergence.
- That is, k^6.
- Then we must include up to 3 factors of the spatial Riemann tensor in the "potential".
- I view this as the central reason why Horava's proposal has any hope of being viable.
- Ultimately, it's just [suitably weighted] power counting.

Horava–Lifshitz gravity:

Let's be a little more systematic about this:

- Introduce a "preferred foliation".
- Take an ADM decomposition:

$$\mathrm{d}s^2 = -N^2 c^2 \mathrm{d}t^2 + g_{ij}\,(\mathrm{d}x^i - N^i \mathrm{d}t)(\mathrm{d}x^j - N^j \mathrm{d}t),$$

- Let κ be a placeholder with the engineering dimensions of momentum and postulate

$$[\mathrm{d}x] = [\kappa]^{-1}; \qquad [\mathrm{d}t] = [\kappa]^{-z}.$$

This really means:
Introduce a quantity Z, with the physical dimensions $[Z] = [\mathrm{d}x]^z/[\mathrm{d}t]$, and then use theoreticians units such that $Z \to 1$.

Horava–Lifshitz gravity:

- Ultimately we shall interpret this quantity Z in terms of the Planck scale, and closely related Lorentz-symmetry breaking scales.
- If one prefers to characterize this quantity Z in terms of a momentum ζ, then $Z = \zeta^{-z+1} c$.
- In order for dimensional analysis to be useful one cannot simultaneously set both $Z \to 1$ and $c \to 1$.
- (Attempting to set both $Z \to 1$ and $c \to 1$ forces both $\mathrm{d}x$ and $\mathrm{d}t$ to be dimensionless, which then renders dimensional analysis impotent.)

Horava–Lifshitz gravity:

In theoretician's ($Z \to 1$) units

$$[N^i] = [c] = \frac{[\mathrm{d}x]}{[\mathrm{d}t]} = [\kappa]^{z-1},$$

and one is free to additionally choose

$$[g_{ij}] = [N] = [1]; \qquad [\mathrm{d}s] = [\kappa]^{-1}.$$

To minimize the algebra, take the volume element to be

$$\mathrm{d}V_{d+1} = \sqrt{g}\, N\, \mathrm{d}^d x\, \mathrm{d}t; \qquad [\mathrm{d}V_{d+1}] = [\kappa]^{-d-z}.$$

The resulting model will, by its very construction, violate Lorentz invariance; the payoff however is greatly improved ultraviolet behaviour for the Feynman diagrams.

Extrinsic curvature:

- This is most conveniently defined to not include any explicit factor of c:

$$K_{ij} = \frac{1}{2N} \left\{ -\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i \right\}.$$

- Then

$$[N^i] = [\mathrm{d}x]/[\mathrm{d}t] = [\kappa]^{z-1}.$$

- Furthermore

$$[K_{ij}] = \frac{[g_{ij}]}{[N][\mathrm{d}t]} = [\kappa]^z.$$

Intrinsic curvature:

For the spatial slices we have

$$[g_{ij}] = [1]; \qquad [\Gamma^i{}_{jk}] = [\kappa]; \qquad [R^i{}_{jkl}] = [\kappa]^2,$$

the key point being

$$[R^{ijkl}] = [\kappa]^2; \quad [\nabla R^{ijkl}] = [\kappa]^3; \quad [\nabla^2 R^{ijkl}] = [\kappa]^4.$$

Kinetic term:

- Consider the quantity

$$\mathcal{T}(K) = g_K \left\{ (K^{ij} K_{ij} - K^2) + \xi K^2 \right\}.$$

(Standard general relativity would enforce $\xi \to 0$.)

- Take the kinetic action to be

$$S_K = \int \mathcal{T}(K) \, \mathrm{d}V_{d+1} = \int \mathcal{T}(K) \sqrt{g} \, N \, \mathrm{d}^d x \, \mathrm{d}t.$$

Kinetic term:

- Then
$$[S_K] = [g_K][\kappa]^{z-d}.$$
- Since the kinetic action is (by definition) dimensionless
$$[g_K] = [\kappa]^{(d-z)}.$$
- The kinetic coupling constant g_K is dimensionless exactly for
$$d = z.$$
- Once we have set $d = z$ to make g_K dimensionless, then provided g_K is positive one can without loss of generality re-scale the time and/or space coordinates to set $g_K \to 1$.

Potential term:

Now consider

$$S_{\mathcal{V}} = \int \mathcal{V}(g)\,\mathrm{d}V_{d+1} = \int \mathcal{V}(g)\,\sqrt{g}\;N\,\mathrm{d}^d x\,\mathrm{d}t,$$

where $\mathcal{V}(g)$ is some scalar built out of the spatial metric and its derivatives.

Then

$$[S_{\mathcal{V}}] = [\mathcal{V}(g)][\kappa]^{-d-z},$$

whence

$$[\mathcal{V}(g)] \to [\kappa]^{d+z}.$$

But to keep the kinetic coupling g_K dimensionless we needed $z \to d$.

Therefore

$$[\mathcal{V}(g)] \to [\kappa]^{2d}.$$

Potential term:

- But $\mathcal{V}(g)$ must be built out of scalar invariants calculable in terms of the Riemann tensor and its derivatives, which tells us it must be constructible from objects of the form

$$\left\{ (\text{Riemann})^d, [(\nabla \text{Riemann})]^2 (\text{Riemann})^{d-3}, \text{etc...} \right\}.$$

- In general, in $d+1$ dimensions this is a long but finite list.
- All of these theories should be well-behaved as quantum field theories.
- All of these theories should have $z = d$ Lifschitz points.

Potential term:

In the specific case $d = 3$ we have

$$[\mathcal{V}(g)] \to [\kappa]^6,$$

and so obtain the short and rather specific list:

$$\Big\{ (\text{Riemann})^3, [\nabla(\text{Riemann})]^2, \\ (\text{Riemann})\nabla^2(\text{Riemann}), \nabla^4(\text{Riemann}) \Big\}.$$

Potential term:

But in 3 dimensions the Weyl tensor automatically vanishes, so we can always decompose the Riemann tensor into the Ricci tensor, Ricci scalar, plus the metric.

Thus we need only consider the much simplified list:

$$\left\{ (\text{Ricci})^3, [\nabla(\text{Ricci})]^2, (\text{Ricci})\nabla^2(\text{Ricci}), \nabla^4(\text{Ricci}) \right\}.$$

But once you look at all the different ways the indices can be wired up this is still relatively messy.

Horava–Lifshitz gravity:

It is roughly at this stage that Horava makes his two great simplifications:

- "projectability";
- "detailed balance".

It is still somewhat unclear whether these are just "simplifying ansatze" or whether they are fundamental to Horava's model.

In particular, Silke Weinfurtner, Thomas Sotiriou, and I have argued that "detailed balance" is not fundamental, and we have started attacking the issue of "projectability".

Horava–Lifshitz gravity:

Eliminating detailed balance:

- Phenomenologically viable Lorentz-violating quantum gravity.
  Thomas Sotiriou, Matt Visser, Silke Weinfurtner. Apr 2009.
  Phys.Rev.Lett.102: 251601, 2009.
  arXiv: 0904.4464 [hep-th]
- Quantum gravity without Lorentz invariance.
  Thomas Sotiriou, Matt Visser, Silke Weinfurtner. May 2009.
  arXiv: 0905.2798 [hep-th]

Projectability:

What is Horava's projectability condition?

$$N(x,t) \to N(t) \quad (\to 1)$$

This is the assertion that the lapse is always trivial (or trivializable).

- In standard general relativity the "projectability condition" can always be enforced locally as a gauge choice;
- Furthermore for "physically interesting" solutions of general relativity it seems that this can always be done globally.
- For instance:
  - for the Schwarzschild spacetime this "projectability condition" holds globally in Painlevé–Gullstrand coordinates;
  - in the Kerr spacetime this condition holds globally (for the physically interesting $r > 0$ region) in Doran coordinates;
  - the FLRW cosmologies also automatically satisfy this "projectability condition".

Projectability:

- For this purely pragmatic reason we decided to put "projectability" off to one side for a while, and first deal with "detailed balance".
- The price we have to pay for enforcing projectability at the level of the action (and before any functional variation), is that the theory we are considering is not necessarily the most general theory with all possible terms of dimension six.
- (But it is still general enough to be a significant generalization with respect to Horava's model).

Detailed balance:

What is Horava's detailed balance condition?

$\mathcal{V}(g)$ is a perfect square.

That is, there is a "pre-potential" $W(g)$ such that:

$$\mathcal{V}(g) = \left(g^{ij} \frac{\delta W}{\delta g_{jk}} \right)^2 .$$

This simplifies some steps of Horava's algebra, it makes other features much worse.

Detailed balance:

Not that this actually helps much when it comes to specific computations, but if you adopt detailed balance then you can apply the process of "stochastic quantization" and derive

$$\dot{g}_{ij} \sim \frac{\delta W}{\delta g^{ij}} + (\text{white noise})_{ij}$$

This is "pretty", but right now does not seem to me to be very useful.

(Potential applications to non-perturbative renormalization.)

Detailed balance:

In particular, if you assume Horava's detailed balance, and try to recover Einstein-Hilbert in the low energy regime, then:

- You are forced to accept a non-zero cosmological constant of the "wrong sign".
- You are forced to accept intrinsic parity violation in the purely gravitational sector.

(The second I could live with, the first will require some mutilation of detailed balance, so we might as well go the whole way and discard detailed balance entirely.)

Discarding detailed balance:

Keep projectability and parity, discard detailed balance, and retain all possible potential terms, eliminating redundant terms using:

- Integration by parts and discarding surface terms;
- Commutator identities;
- Bianchi identities;
- Special relations appropriate to 3 dimensions.
  (Weyl vanishes; properties of Cotton tensor.)

Discarding detailed balance:

There are only five independent terms of dimension $[\kappa]^6$:

$$R^3, \quad R R^i{}_j R^j{}_i, \quad R^i{}_j R^j{}_k R^k{}_i; \quad R\, \nabla^2 R, \quad \nabla_i R_{jk}\, \nabla^i R^{jk}.$$

These terms are all marginal (renormalizable) by power counting.

In Horava's article only one particular linear combination of these five terms is considered:

Phrased in terms of the Cotton tensor, Horava considers the single (perfect square) $[\kappa]^6$ term $C^i{}_j C^j{}_i$.

Discarding detailed balance:

Add all possible lower-dimension terms (relevant operators, super-renormalizable by power-counting):

$$[\kappa]^0 : \quad 1; \qquad [\kappa]^2 : \quad R; \qquad [\kappa]^4 : \quad R^2; \quad R^{ij} R_{ij}.$$

This now results in a potential $\mathcal{V}(g)$ with nine terms and nine independent coupling constants.

In contrast, Horava chooses a potential $\mathcal{V}(g)$ containing six terms with only three independent coupling constants, of the form

$$(\tilde{g}_2 \,\text{Cotton} + \tilde{g}_1 \,\text{Ricci} + \tilde{g}_0 \,\text{metric})^2.$$

Assembling the pieces:

$$S = \int [\mathcal{T}(K) - \mathcal{V}(g)] \sqrt{g}\, N \, \mathrm{d}^3x \, \mathrm{d}t,$$

with

$$\begin{aligned} \mathcal{V}(g) \;=\;& g_0\,\zeta^6 + g_1\,\zeta^4\,R + g_2\,\zeta^2\,R^2 + g_3\,\zeta^2\,R_{ij}R^{ij} \\ & + g_4\,R^3 + g_5\,R(R_{ij}R^{ij}) + g_6\,R^i{}_j R^j{}_k R^k{}_i \\ & + g_7\,R\nabla^2 R + g_8\,\nabla_i R_{jk}\,\nabla^i R^{jk}, \end{aligned}$$

where we have introduced suitable factors of ζ to ensure the couplings g_a are all dimensionless.

Now assuming $g_1 < 0$, we can without loss of generality re-scale the time and space coordinates to set *both* $g_K \to 1$ and $g_1 \to -1$.

Assembling the pieces:

The Einstein–Hilbert piece of the action is now

$$S_{\mathrm{EH}} = \int \left\{ (K^{ij} K_{ij} - K^2) + \zeta^4 R - g_0\, \zeta^6 \right\} \sqrt{g}\; N\; \mathrm{d}^3 x\; \mathrm{d}t,$$

and the "extra" Lorentz-violating terms are

$$\begin{aligned} S_{\mathrm{LV}} &= \int \Big\{ \xi\, K^2 - g_2\, \zeta^2\, R^2 - g_3\, \zeta^2\, R_{ij} R^{ij} \\ &\quad - g_4\, R^3 - g_5\, R (R_{ij} R^{ij}) - g_6\, R^i{}_j R^j{}_k R^k{}_i \\ &\quad - g_7\, R \nabla^2 R - g_8\, \nabla_i R_{jk}\, \nabla^i R^{jk} \Big\} \sqrt{g}\; N\; \mathrm{d}^d x\; \mathrm{d}t. \end{aligned}$$

Physical units:

- While these $Z \to 1$ units have been most useful for power counting purposes, when it comes to phenomenological confrontation with observation it is much more useful to adapt more standard "physical" ($c \to 1$) units, in which $[\mathrm{d}x] = [\mathrm{d}t]$.
- The transformation to physical units is most easily accomplished by setting $(\mathrm{d}t)_{Z=1} \to \zeta^{-2} (\mathrm{d}t)_{c=1}$.

Physical units:

In these physical units the Einstein–Hilbert piece of the action becomes

$$S_{\mathrm{EH}} = \zeta^2 \int \left\{ (K^{ij} K_{ij} - K^2) + R - g_0 \, \zeta^2 \right\} \sqrt{g} \; N \, \mathrm{d}^3 x \, \mathrm{d}t,$$

and the "extra" Lorentz-violating terms become

$$\begin{aligned} S_{\mathrm{LV}} \;=\; \zeta^2 \int \Big\{ & \xi \, K^2 - g_2 \, \zeta^{-2} \, R^2 - g_3 \, \zeta^{-2} \, R_{ij} R^{ij} \\ & - g_4 \, \zeta^{-4} \, R^3 - g_5 \, \zeta^{-4} \, R(R_{ij} R^{ij}) \\ & - g_6 \, \zeta^{-4} \, R^i{}_j R^j{}_k R^k{}_i - g_7 \, \zeta^{-4} \, R \nabla^2 R \\ & - g_8 \, \zeta^{-4} \, \nabla_i R_{jk} \, \nabla^i R^{jk} \Big\} \sqrt{g} \; N \, \mathrm{d}^d x \, \mathrm{d}t. \end{aligned}$$

Physical units:

- From the normalization of the Einstein–Hilbert term:

$$(16\pi G_{\mathrm{Newton}})^{-1} = \zeta^2; \qquad \Lambda = \frac{g_0\,\zeta^2}{2};$$

  so that ζ is identified as the Planck scale.
- The cosmological constant is determined by the free parameter g_0, and observationally $g_0 \sim 10^{-123}$ (renormalized after including vacuum energy contributions).
- In particular, the way we have set this up we are free to choose the Newton constant and cosmological constant independently (and so to be compatible with observation).

Physical units:

- The Lorentz violating term in the kinetic energy leads to an extra scalar mode for the graviton, with fractional $O(\xi)$ effects at all momenta.
- Phenomenologically, this behaviour is potentially dangerous and should be carefully investigated.
- The various Lorentz-violating terms in the potential become comparable to the spatial curvature term in the Einstein–Hilbert action for physical momenta of order

$$\zeta_{\{2,3\}} = \frac{\zeta}{\sqrt{|g_{\{2,3\}}|}}; \qquad \zeta_{\{4,5,6,7,8\}} = \frac{\zeta}{\sqrt[4]{|g_{\{4,5,6,7,8\}}|}}.$$

Physical units:

- The Planck scale ζ is divorced from the various Lorentz-breaking scales $\zeta_{\{2,3,4,5,6,7,8\}}$.
- We can drive the Lorentz breaking scale arbitrarily high by suitable adjustment of the dimensionless couplings $g_{\{2,3\}}$ and $g_{\{4,5,6,7,8\}}$.
- Grisha Volovik has been asserting for many years that the Lorentz-breaking scale should be much higher than the Planck scale.
- This model naturally implements that idea.

Horava–Lifshitz gravity — Problems and pitfalls:

Where are the bodies buried?

- Projectability: This yields a spatially integrated Hamiltonian constraint rather than a super-Hamiltonian constraint.
- Prior structure: Is the preferred foliation "prior structure"? Or is it dynamical?
- Scalar graviton: As long as $\xi \neq 0$ there is a spin-0 scalar graviton in addition to the spin-2 tensor graviton.

All three of these issues have the potential to lead to violent conflicts with empirical reality.

Graviton propagator linearized around flat space:

- Spin-0 scalar graviton:

$$\left(1-\frac{3}{2}\xi\right)\ \ddot{h}=\xi\left\{\frac{1}{2}\partial^2+\left(4g_2+\frac{3}{2}g_3\right)\partial^4+\left(-4g_7+\frac{3}{2}g_8\right)\partial^6\right\}h.$$

(The case $\xi = 0$ is special; requiring a separate analysis.)

- Spin-2 tensor graviton:

$$\ddot{\widetilde{H}}_{ij}=\left[\partial^2-g_3\partial^4-g_8\partial^6\right]\widetilde{H}_{ij}.$$

- Trans–Bogoliubov dispersion relation.

Graviton propagator linearized around flat space:

The spin-0 scalar graviton is potentially dangerous:

- Binary pulsar?
  (Extra energy loss mechanism.)
- PPN physics?
- Eötvös experiments?

Lots of careful thought needed...

FLRW spacetimes:

First Friedmann equation:

$$\left(1-\frac{3}{2}\xi\right)\;\frac{\dot{a}^2}{a^2}=\frac{\mathcal{V}(a)}{6}+\frac{8\pi\,G_{\rm N}\,\rho}{3}.$$

Second Friedmann equation:

$$-\left(1-\frac{3}{2}\xi\right)\;\frac{\ddot{a}}{a}=+\frac{1}{2}\left(1-\frac{3}{2}\xi\right)\frac{\dot{a}^2}{a^2}-\frac{1}{12a^2}\frac{{\rm d}[\mathcal{V}(a)\,a^3]}{{\rm d}a}+4\pi\,G_N\,p.$$

Third Friedmann equation:

$$-\left(1-\frac{3}{2}\xi\right)\;\frac{\ddot{a}}{a}=-\frac{1}{12a}\frac{{\rm d}[\mathcal{V}(a)a^2]}{{\rm d}a}+\frac{4\pi\,G_{\rm N}}{3}(\rho+3p).$$

Potential ($k\in\{-1,0,+1\}$):

$$\mathcal{V}(a)=2\Lambda+\frac{6k}{a^2}+\frac{12(3g_2+g_3)\zeta^{-2}k^2}{a^4}+\frac{24(9g_4+3g_5+g_6)\zeta^{-4}k}{a^6}.$$

FLRW spacetimes:

That is, the (generalized) Horava–Lifshitz model naturally provides:

- Dark radiation;
- Dark stiff matter.

From the Hamiltonian/ super-Hamiltonian distinction, can potentially get:

- Dark dust.

Summary:

- Still a very active field...
- (Initial feeding frenzy somewhat subsided)...
- Real physics challenges remain...
- Worth withholding judgment for another 6 months or so...

End: