

Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui



Black Holes:
Theory and Observation

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Basics:



- The idea of a “black hole” pre-dates Einstein’s general relativity by 132 years.
- Blame it on:
 - Reverend John Mitchell (1783);
 - Peter Simon Laplace (1798).
- Newton’s physics is enough to get the basic idea across.
- I will use three formulas — that’s all.

Dark stars:



- In Newton's nonrelativistic theory of gravity the escape velocity from the surface of a star or planet is:

$$\frac{1}{2}v_{\text{escape}}^2 = \frac{G_{\text{Newton}} M}{R}$$

- Therefore, if

$$\frac{1}{2}c^2 < \frac{G_{\text{Newton}} M}{R}$$

then light cannot escape and we have a "dark star".

- That is, anything sufficiently "heavy" and "small" will be "dark".
- The physics is more complicated, but the same basic idea holds in Einstein's relativistic theory of gravity (the general relativity).

Laplace:



A luminous star, of the same density as the earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its [light] rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible.

Peter Simon Laplace (1798).

Einstein and Schwarzschild:



- The mathematics of Einstein's gravity [the general relativity] is much more complicated than the mathematics of Newton's gravity.
- History:
 - Einstein (1915): theory formulated.
 - Schwarzschild (1916): first “exact solutions”.
- Schwarzschild radius:

$$R_S = \frac{2G_{\text{Newton}} M}{c^2}$$

- If the actual radius of the star is less than the Schwarzschild radius, then nothing escapes — not even light.

Black holes:



- It took about 45 years for the physicists/mathematicians to fully understand the Schwarzschild solution.
- By 1960 theoretical and mathematical issues were getting a little clearer.
- Phrases like “dark star” and “collapsed star” [“collapsar”] fell into disuse; and the phrase “black hole” was coined.
- Blame John Archibald Wheeler for the terminology. [He’s also responsible for “worm-hole”, and a number of other phrases.]
- By Y2K most astrophysicists [more than 99%] were firmly convinced that:
“black holes have been observed”.

Do black holes exist?



So do black holes really truly exist?

- Theory says yes; they are there as mathematical solutions of the field equations.
- Physics says yes; the overall experimental evidence for general relativity is very good.
- Astrophysics says yes; there is no other way to make sense of some of the observations.
- Direct observation says “yes”; there are certainly things out there that are small, heavy, and dark — at a certain stage you just have to go with the preponderance of evidence.

Observation:



- What does it mean to “observe” a black hole?
 - You cannot “see” it because light cannot escape.
 - You need indirect tests.
Look for something that is both:
 - Compact;
- $$\frac{2G_{\text{Newton}} M}{R c^2} = \text{“large”}.$$
- Dark.
- There are three main areas to start looking.

Candidates:



- The three standard places to look are:
 - Galactic cores;
 - * new; improved; great data.
 - Burnt out stars/ supernova remnants;
 - * traditional; good data; gravity waves?
 - Micro black holes;
 - * not seen yet; quantum physics?
- Other nonstandard places could surprise us.
- Intermediate mass black holes?

Galactic cores:



- Look at the central 2×2 arc-second square centered on the radio source Sagittarius A* at the center of our galaxy.
- That's $\frac{1}{1000} \times \frac{1}{1000}$ the apparent size of the full moon.
- There's a lot of dust in the way, but using infrared we can make out a few individual stars in very tight orbits around the center of our galaxy.
- The orbits are so tight that we can see the stars move over the lifetime of a typical graduate student.
- Whatever they are orbiting around has to be heavy — big time heavy.

Star S2:



- Orbital Period: 15.2 years.
- Orbital speed:
5000 km/s — 2% lightspeed.
- Peri-whatever: 17 light hours = 125 AU.
- Mass of whatever it is orbiting:
more than 2,500,000 mass of our Sun.
- That's a lot of mass in a small, dark, region.
- Without a black hole, we have no sensible way of doing this within the rules of physics.

Logic:



- Note the argument is indirect — you establish mass and radius; and try to fit those observations using what you know about stellar structure and nuclear physics.
- All attempts at doing so lead to models that collapse to black holes.
- The models are good enough that [almost all] people are convinced.
 - “Orbital periods as short as 15 years clinch the case for a supermassive black hole at the Galaxy’s heart.”
- Similar things seem to be happening in other galaxies.

Supernova remnants:



- The other standard place to look is for burnt-out stars, especially supernova remnants.
- If the mass of the burnt-out remnant is greater than about twice the mass of the Sun, then it cannot support itself once the nuclear fires have gone out — it will collapse to form a black hole.
- Proving this requires some messy nuclear physics. [[Chandrasekhar limit](#)].
- Smaller burnt-out remnants can support themselves; they just become burnt-out cinders of stars — they go through a white-dwarf phase and slowly cool down.

What to look for:



- Look for X-rays or gamma rays.
- Look for orbital motion.
- If you see both; chances are it is a binary system with an “accretion disk” .
- The X-rays/ gamma rays are coming from the “accretion disk”; not from the black hole.
- Then the hard work starts:
 - Calculate masses/sizes;
 - Check for ADAF;
[advection dominated accretion flow].

ADAF:



Advection Dominated Accretion Flow:

Look for two things [1 positive/1 negative]:

- Radiation from the accretion disk;
— the gas gets hot as it is pulled in.
- Lack of “bremsstrahlung”
[braking radiation];
— no evidence of the gas hitting a stellar surface;
— indicates that the gas is being sucked into an event horizon.
— this is critically important evidence.

Plus:

- masses from orbital parameters;
- sizes from time variability.

Death spirals:



- General relativity [Schwarzschild solution] has an innermost stable circular orbit [ISCO] at $3/2$ the Schwarzschild radius.
- Nothing like this in Newton's gravity.
- Every now and then we “see” hot clumps of gas whizzing around the black hole; slowly losing energy to friction, falling to a lower orbit, and so speeding up... until, that is, they hit the innermost stable circular orbit, at which stage: they softly and quietly vanish away...
- That is, we can see the “innermost stable circular orbit” and it is where general relativity predicts.
- For the snark was a boojum, you see.

Buchdahl–Bondi bound:



- If the radius is less than the Schwarzschild radius it's definitely a black hole; by definition.
- If the radius is less than $3/2$ the Schwarzschild radius there's an ISCO; so seeing an ISCO tells you it's close to forming a black hole.
- Surprise: You cannot get the stellar radius arbitrarily close to the Schwarzschild radius.
- A technical calculation yields the Buchdahl–Bondi bound; independent of nuclear physics.
- If the radius is less than $9/8$ the Schwarzschild radius the star is guaranteed to be unstable; it must collapse.

Gravity waves:



- Binary systems also have other ways of losing energy apart from accretion disks.
- Strictly speaking: There are no stable orbits in general relativity since you always lose some energy to gravity waves.
- Waving an electric charge around generates electromagnetic waves.
- Waving a mass around generates gravity waves.
- Jupiter in its orbit around our Sun, is constantly radiating about 80W in the form of gravity waves — a minuscule undetectible amount.

Plunge and merger:



- But for two heavy stars in a tight binary orbit the energy loss to gravity waves can be substantial.
- We have already seen indirect evidence from orbital changes in the Hulse–Taylor binary pulsar.
- The binary system has an ISCO, which is now a complicated function of both masses.
- Once the binary loses enough energy to hit the ISCO there is a rapid and violent “plunge and merger” with the two stars hitting head-on at a decent fraction of light-speed.

Chirps:



- Star/star mergers of this type should easily produce black holes; plus a big splash.
- Star/black-hole mergers will produce substantially bigger black holes; and lots of gravity waves from the merger.
- Black-hole/black-hole mergers will produce substantially bigger black holes; and an enormous amount of gravity waves from the merger [plus almost no light].
- We are looking: one gravity wave observatory is coming online [LIGO]; another is in the serious planning stage [LISA].
- The final 15 minutes before the merger should produce a characteristic “chirp” — a brief pulse of rapidly increasing frequency.

Micro black holes:



- We expect “primordial black holes” to be out there; micro black holes left over from the big bang.
- Micro meaning “mass of mount Everest” or smaller.
- We’re still looking; have not found any yet.
- Micro black holes are important as tests of the quantum/ classical interface.
- Stephen Hawking:
 - Once quantum mechanics is included, black holes are not entirely black.

Hawking radiation:



- Quantum fluctuations can [in a suitably perverted sense] leak across the event horizon.
- More precisely:
 - Quantum fluctuations let you borrow energy from the vacuum, provided you pay it back quickly.
 - Quantum fluctuations permit temporary creation of particles from the vacuum; provided some of the particles have negative energy.
 - This is happening all the time; which is why the particle physicists have such a hard time dealing with the quantum vacuum.

Curved-space QFT:



- Sometimes the negative energy falls into the black hole; letting the positive energy quantum fluctuation escape...
- After a disgustingly foul technical calculation involving quantum field theory in curved spacetime:
 - Black holes should radiate an [almost exact] thermal spectrum with a temperature proportional to the “surface gravity” at the event horizon.
 - As the black hole radiates, it loses mass, it gets smaller, the surface gravity goes up, and it radiates even more.
 - “Black hole explosions?”

Explosions:



- We are still looking for the black hole explosions.
- Have not found them yet.
- But Hawking radiation really has to be there! — the theoretical input is so basic and fundamental.
- Lots of theory papers; no experiments; few observations.
- Part of my own research has to do with the possibility of simulating Hawking radiation [and more generally, curved spacetime] in various condensed matter systems.

Conclusions:



- Black holes exist [at the 99%+ level].
- The only significant issue is whether something goes drastically wrong at the event horizon itself — we certainly have very good evidence all the way up to the event horizon.
- The ADAFs strongly suggest the event horizon is doing what we expect — acting as a one-way membrane.
- The number of even marginally acceptable alternatives to the general relativity can be counted on the fingers of one severely mutilated hand.