Victoria University of Wellington
Te Whare Wānanga o te Ūpoko o te Ika a Māui
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--- Gravity ---
Everybody knows what it is, but nobody quite understands it...

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New Plymouth
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Abstract:

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From Newton to Einstein to Hawking, physicists have been developing and extending our ideas of what gravity is and how it should be described.

In particular, physicists have spent the last 50 years trying to merge quantum physics with Einstein's ideas on gravity.

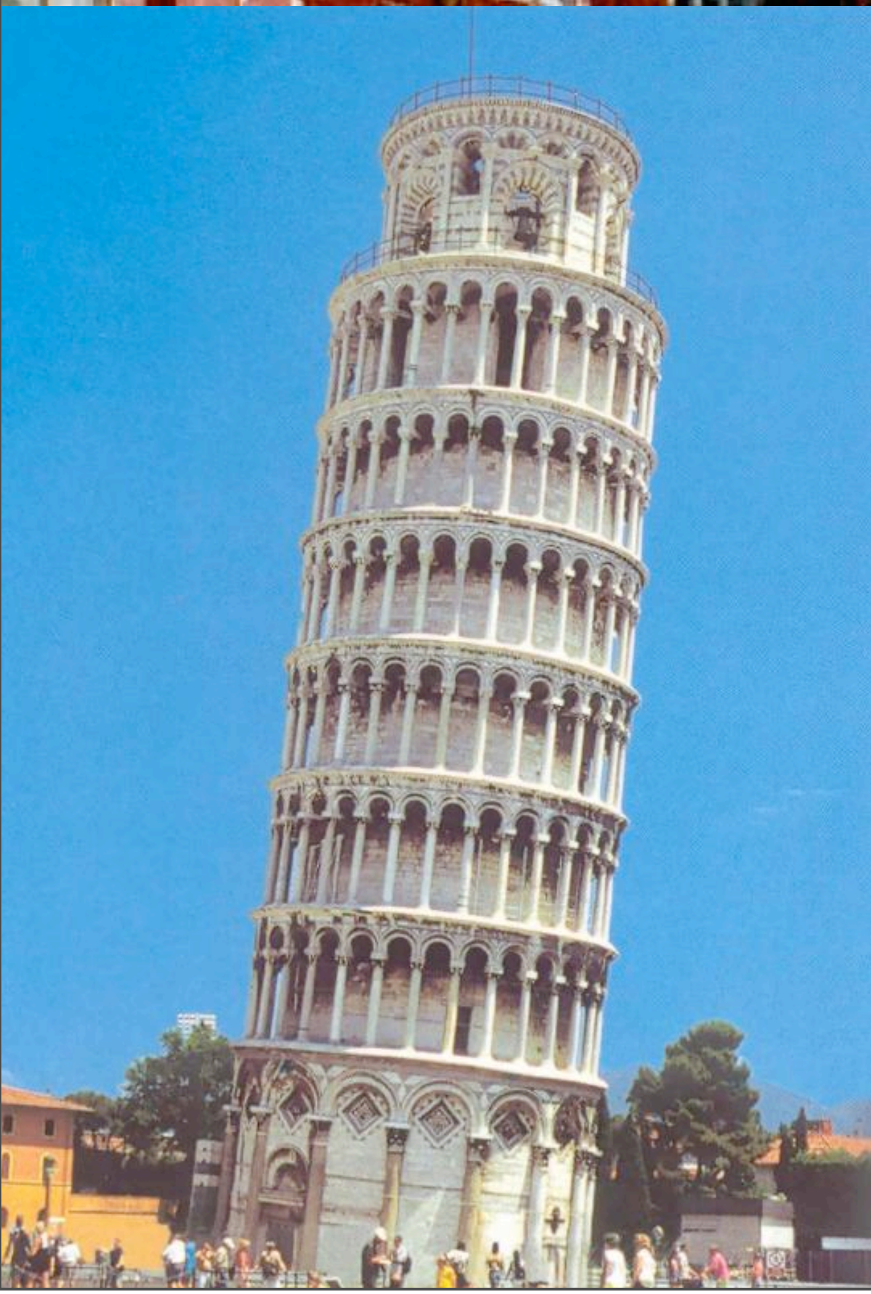
What on earth is going on?

Why are we interested?

Let's look at the possibilities...

The first really quantitative results on gravity were due to Galileo...

If we neglect air resistance, and this is sometimes a good approximation, then gravity is ***universal***



Near the surface of the earth, the acceleration due to gravity is universal and approximately:

g ~ 9.81 metres/sec/sec

g ~ 32 feet/sec/sec

The leaning tower story is probably nonsense...

The pendulum story is however, definitely historical fact...

(Something to do when the sermon is in full swing...)

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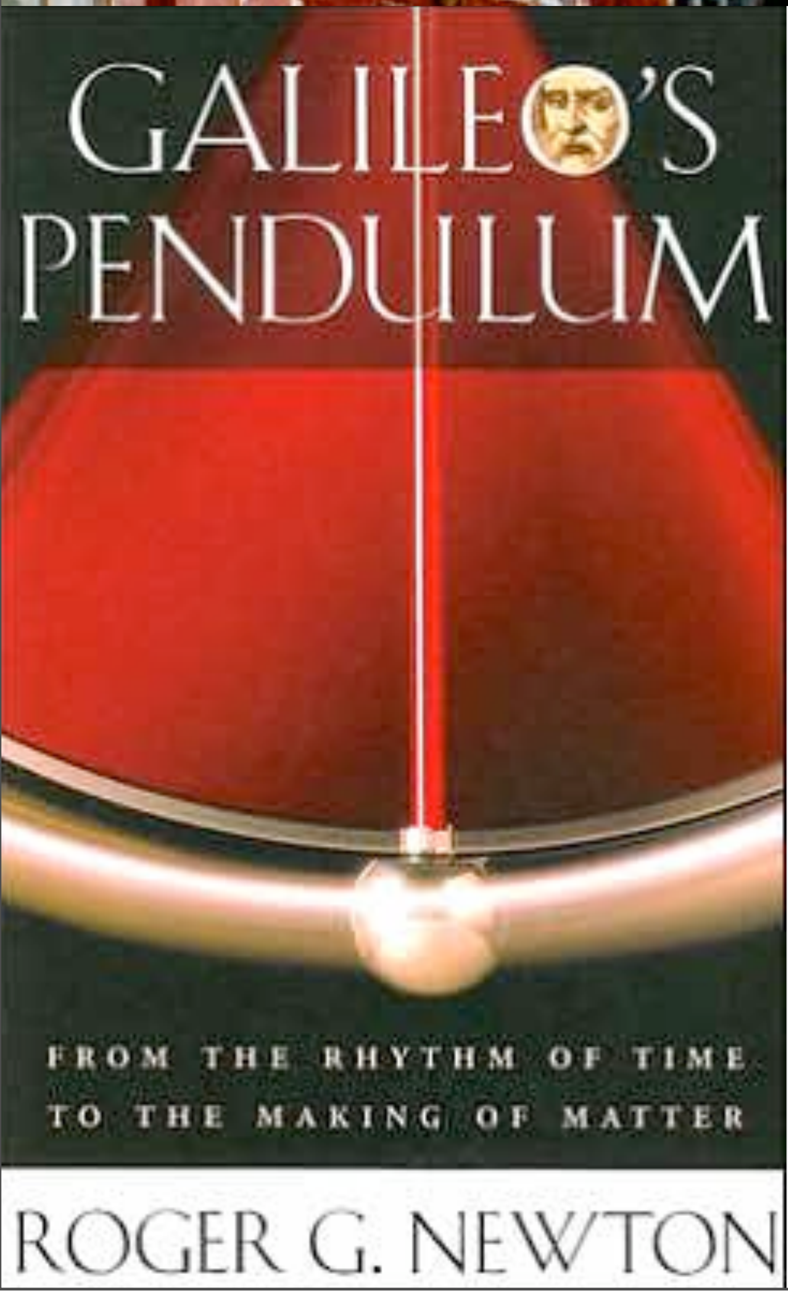
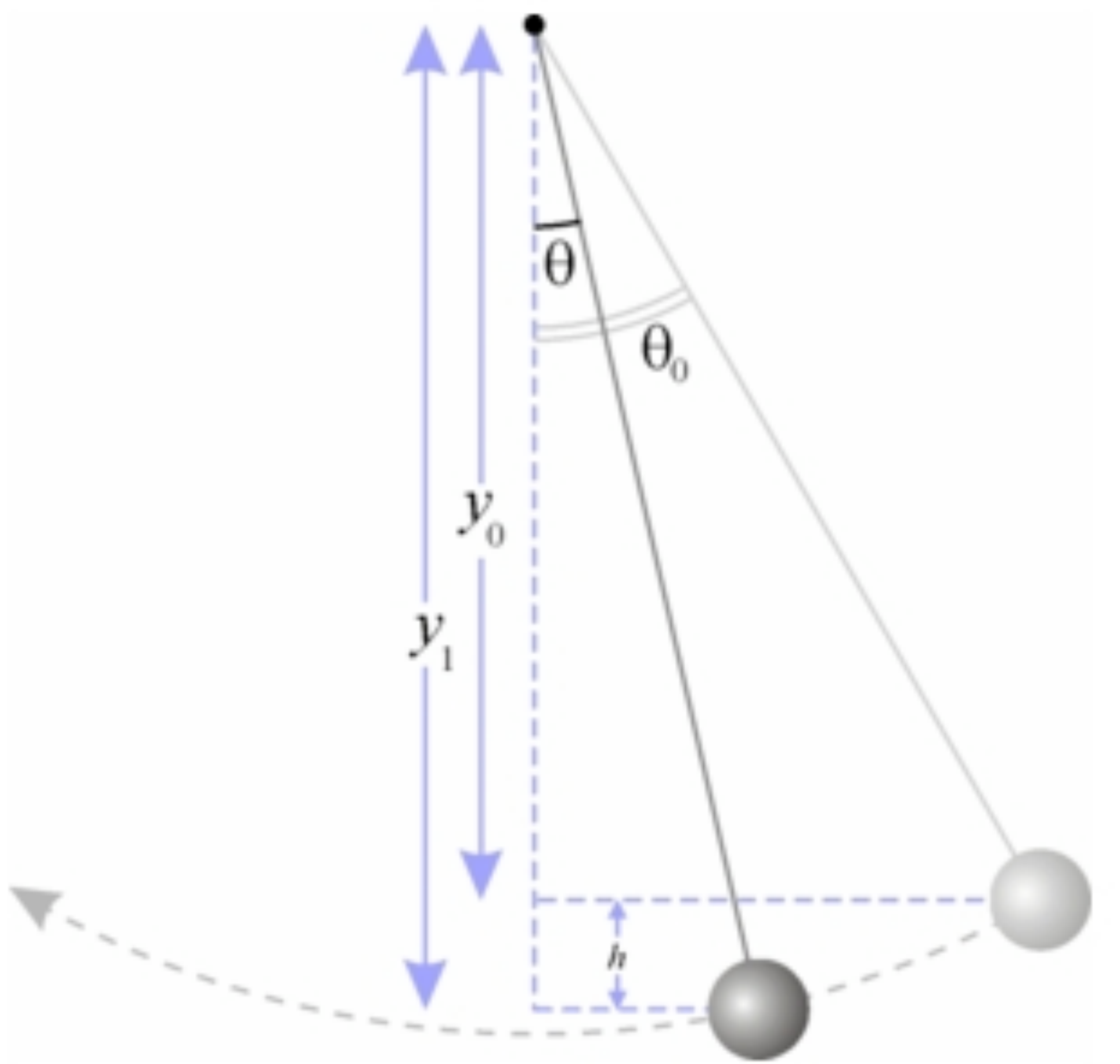


By timing the pendulum,
the chandelier of the
cathedral at Pisa,
and noting that the
time for a single beat
was independent of how
much it was swinging,
Galileo could "see"
the beginnings of
mathematical physics...



This inspiration then led Galileo to carry out some experiments...

Galileo's experiments:

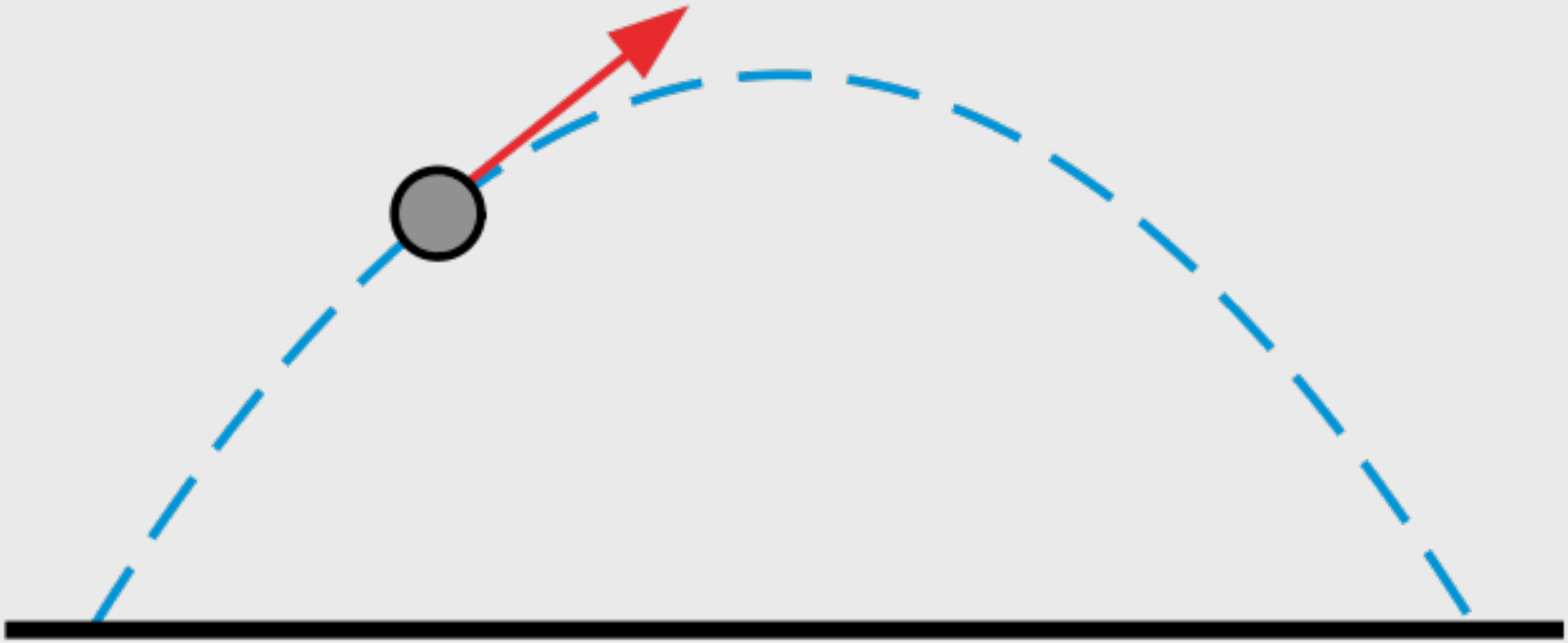
- Pendulums nearly return to their release heights.
- All pendulums eventually come to rest, with the lighter ones coming to rest faster.
- The period is independent of the bob weight.
- The period is independent of the amplitude.
- The square of the period varies directly with the length.

The period is independent of the bob material...

Technologically, this led to the pendulum clock, eventually to the “grandfather clock”...

Physically, this led to the notion of gravity as a force of universal acceleration... (at least near the surface of the earth)

Mathematically, it led to all those “constant acceleration” problems... (ask your teenage offspring, provided they are taking calculus and/or physics)...



Neglecting air resistance is a reasonably good approximation for cannon balls

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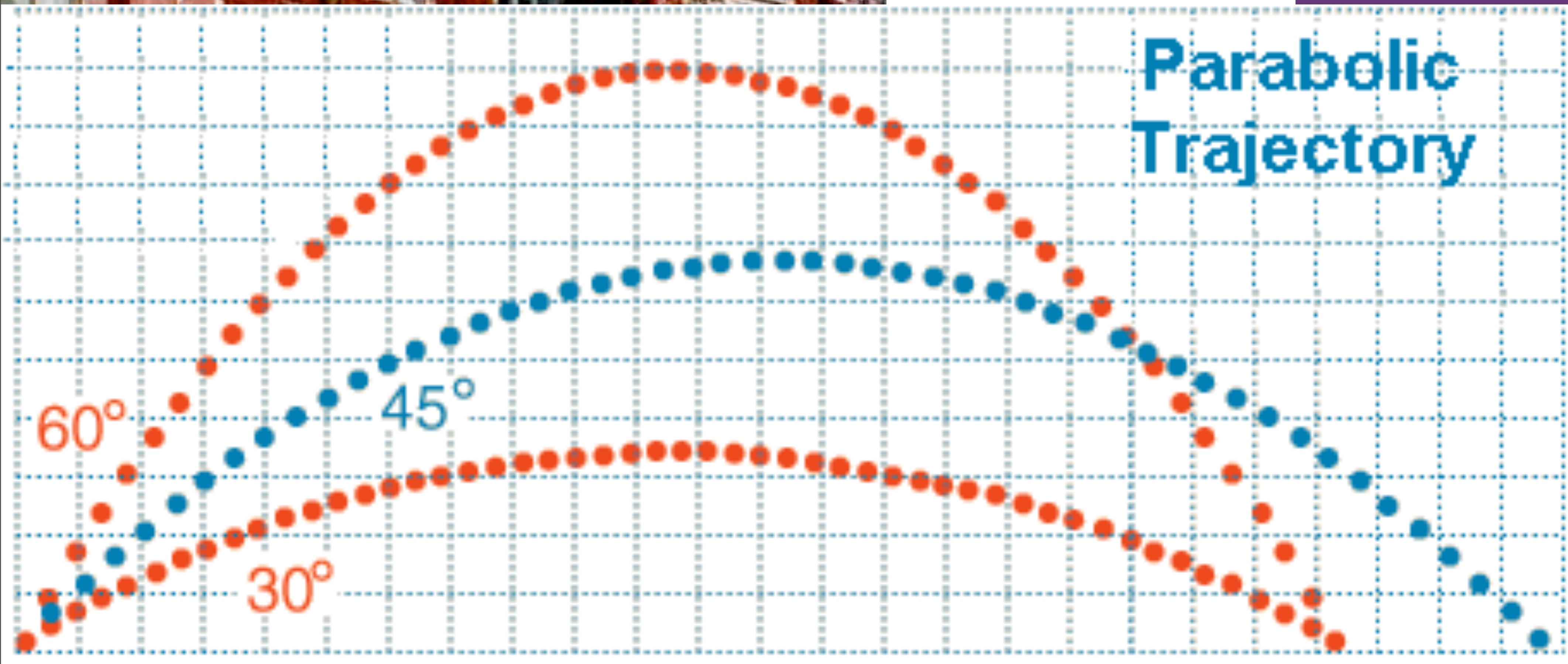


If it is safe to neglect air resistance, then 45 degrees gives you maximum range...

Next came Newton

Inverse-square law

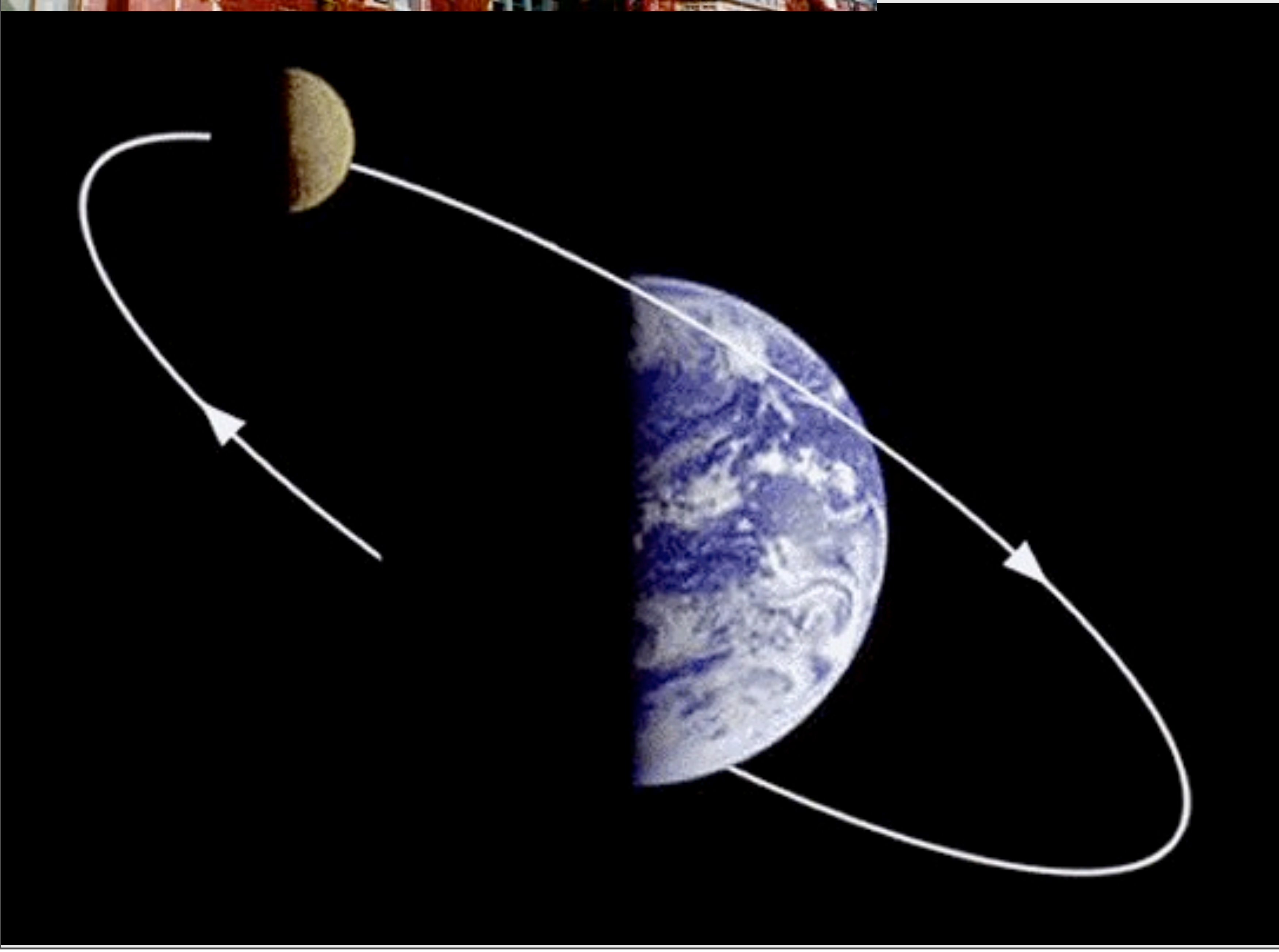
Gravity near the surface of the earth is intimately related to the orbit of the moon around the earth...

"Nothing yet. ...How about you, Newton?"

Inverse-square law

The acceleration due to gravity falls off as the square of the distance...

$$g(r) = g_S \left(\frac{r_S}{r}\right)^2$$



Prediction based on inverse square law:

$$g(r) = g_S \left(\frac{r_S}{r}\right)^2$$

$$\text{Radius of earth} \ = 6\ 371 \text{ km}$$

$$\text{Radius of moon's orbit} \ = 384\ 748 \text{ km}$$

$$\text{acceleration due to gravity at earth's surface} = 9.81 \text{ metres/sec/sec}$$

$$\text{Predicted acceleration of moon} \ = (9.81 \text{ metres/sec/sec}) \times \left(\frac{6\ 371 \text{ km}}{384\ 748 \text{ km}}\right)^2$$

$$\text{Predicted acceleration of moon} \ = 0.00269 \text{ metres/sec/sec}$$

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Observation:

$$\text{Period of moon's orbit } = 27.321 \text{ days} = 2.351 \times 10^6 \text{ sec}$$

$$\text{Speed of moon in its orbit } = 1.023 \text{ km/sec} = 1023 \text{ metres/sec}$$

$$\text{Observed acceleration of moon } = \frac{v^2}{R} = \frac{(1023 \text{ metres/sec})^2}{384\ 748\ 000 \text{ km}}$$

$$\text{Observed acceleration of moon } = 0.00272 \text{ metres/sec/sec}$$

versus

$$\text{Predicted acceleration of moon } = 0.00269 \text{ metres/sec/sec}$$

(Not bad for a crude approximation using circular orbits)

"As above, so below"

"As below, so above"

The key point is that Newton realized that gravity, here on earth, is the same as gravity out in the solar system...

This led him to planetary orbits, and "celestial mechanics"...

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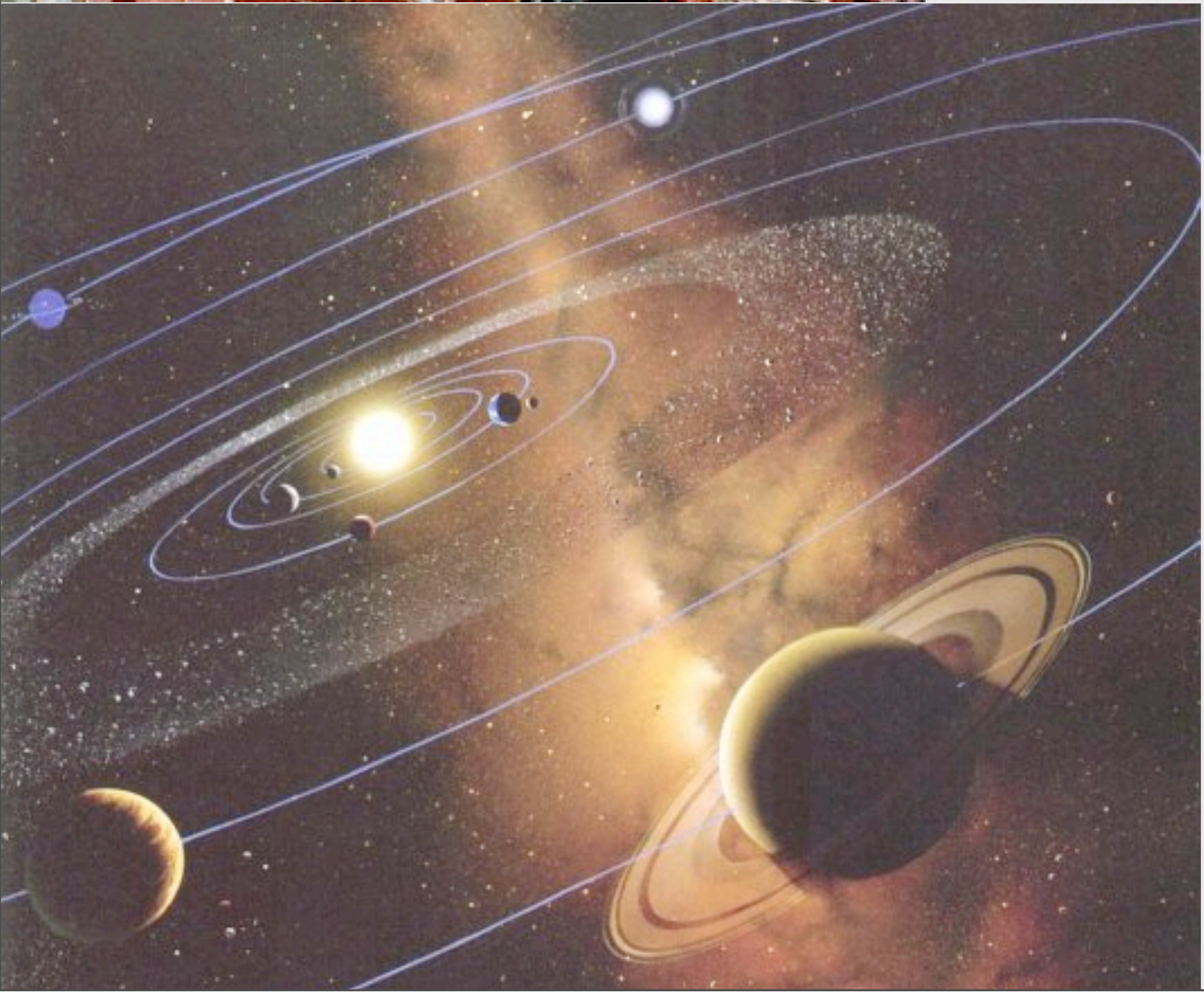

Newton's gravity is an excellent approximation for solar system dynamics...

(Mercury, Pioneer anomaly...)

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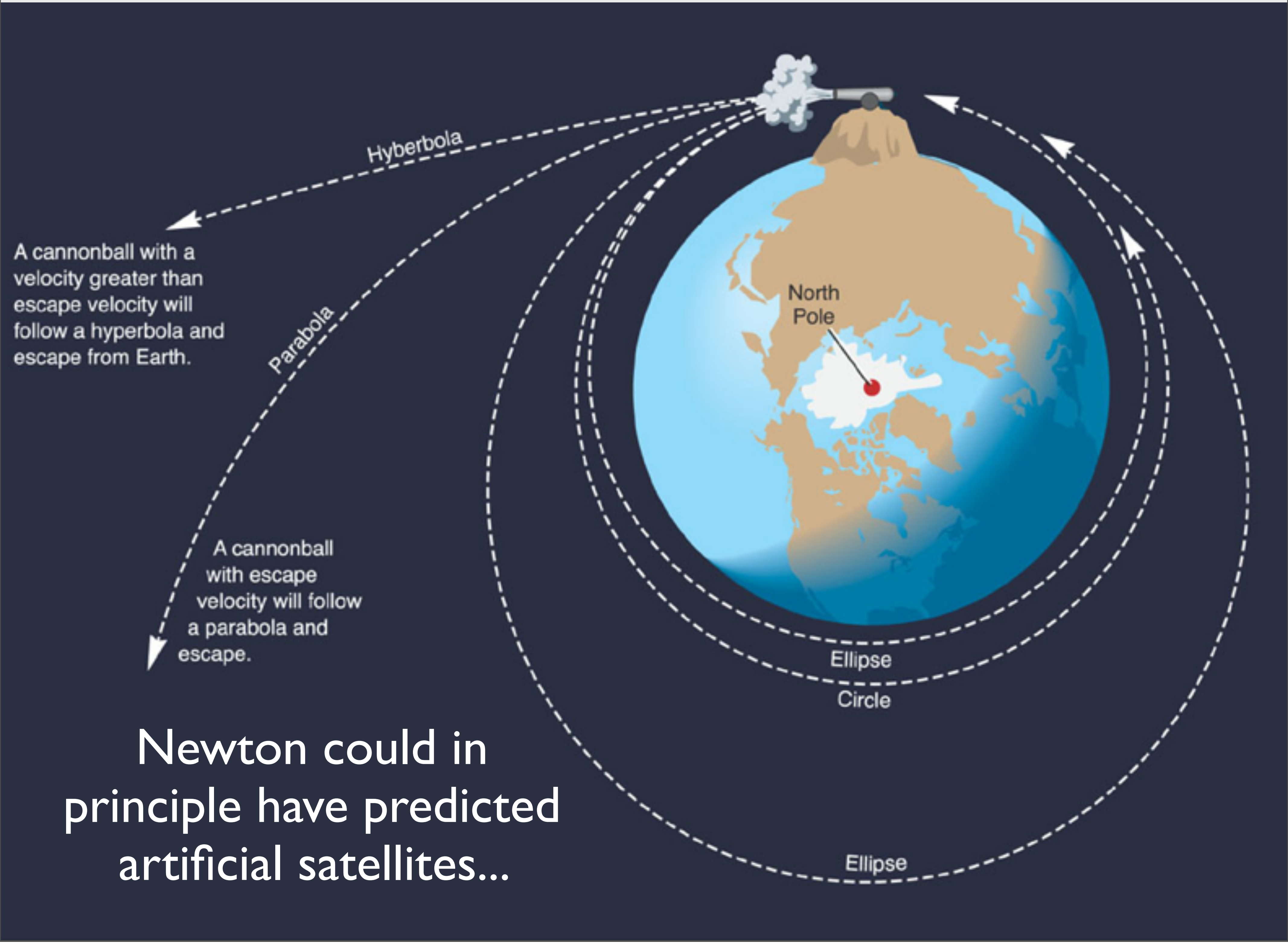
Solar system orbits,
planets, asteroids, comets, etc,
are
circles, ellipses, parabolas,
or hyperbolas...

(at least to a first approximation,
until you look at the way
planets, asteroids, comets, etc
interact with each other...)



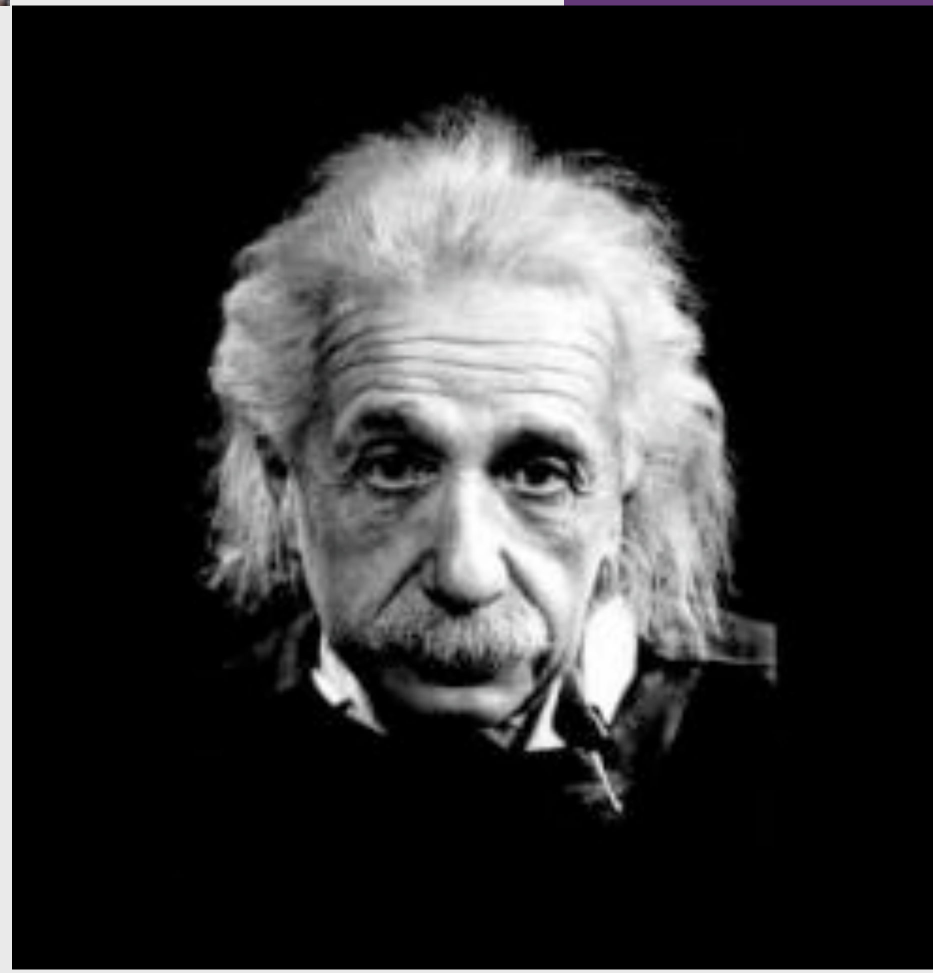
Newton could in principle have predicted artificial satellites...

Curved space-time:

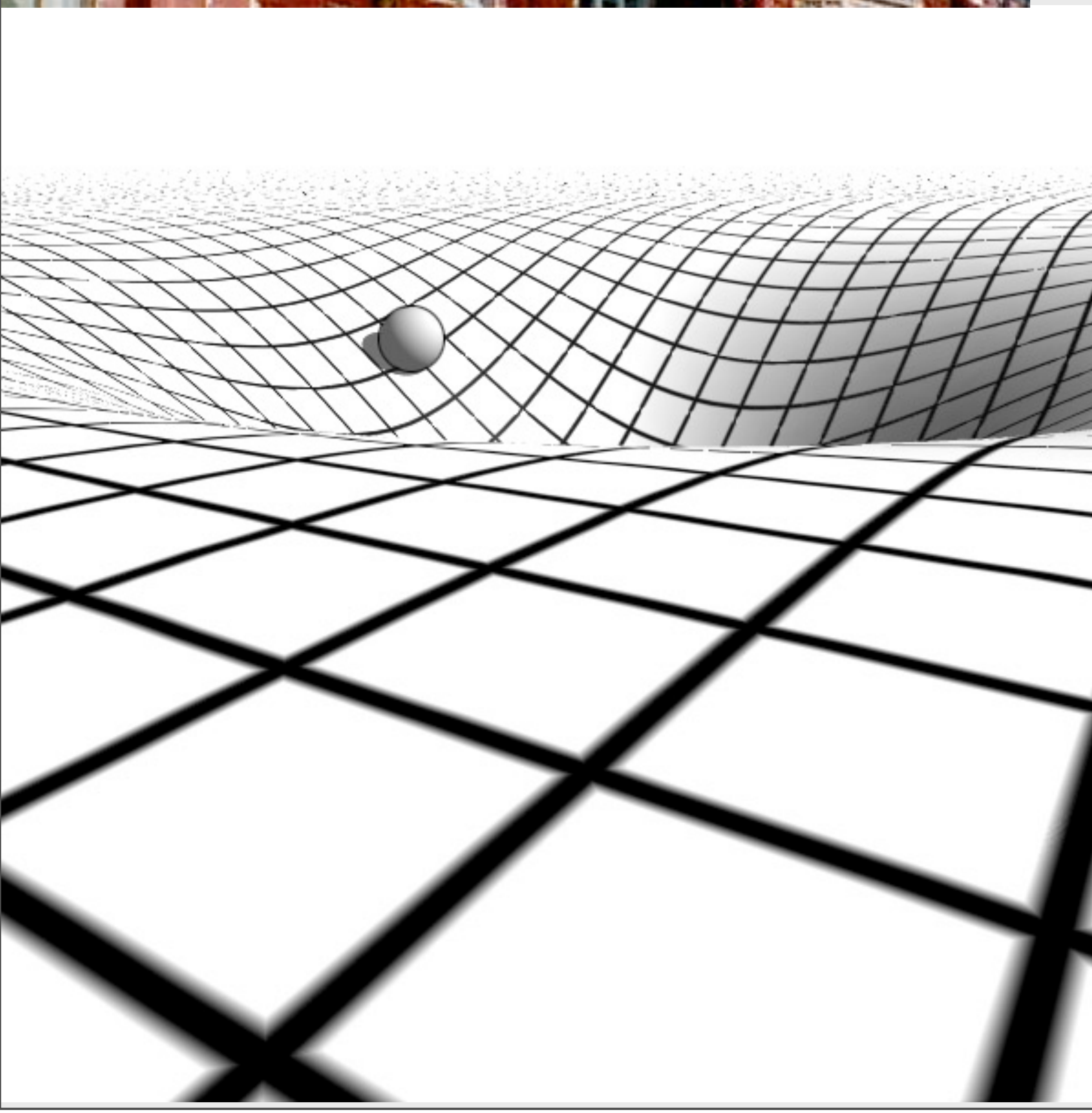
Enter Einstein...

Euclid’s geometry is not exactly “real world”...

(Euclid’s geometry is correct mathematics, but that does not make it “real world physics”...)



To more accurately describe physical reality you need Riemann’s curved space+time...



Space tells matter how to move...

Matter tells space how to curve...

$$G_{ab} = 8\pi G_N \, T_{ab}$$

Looks innocent, doesn't it?

$$G_{ab} = 8\pi G_N \, T_{ab}$$

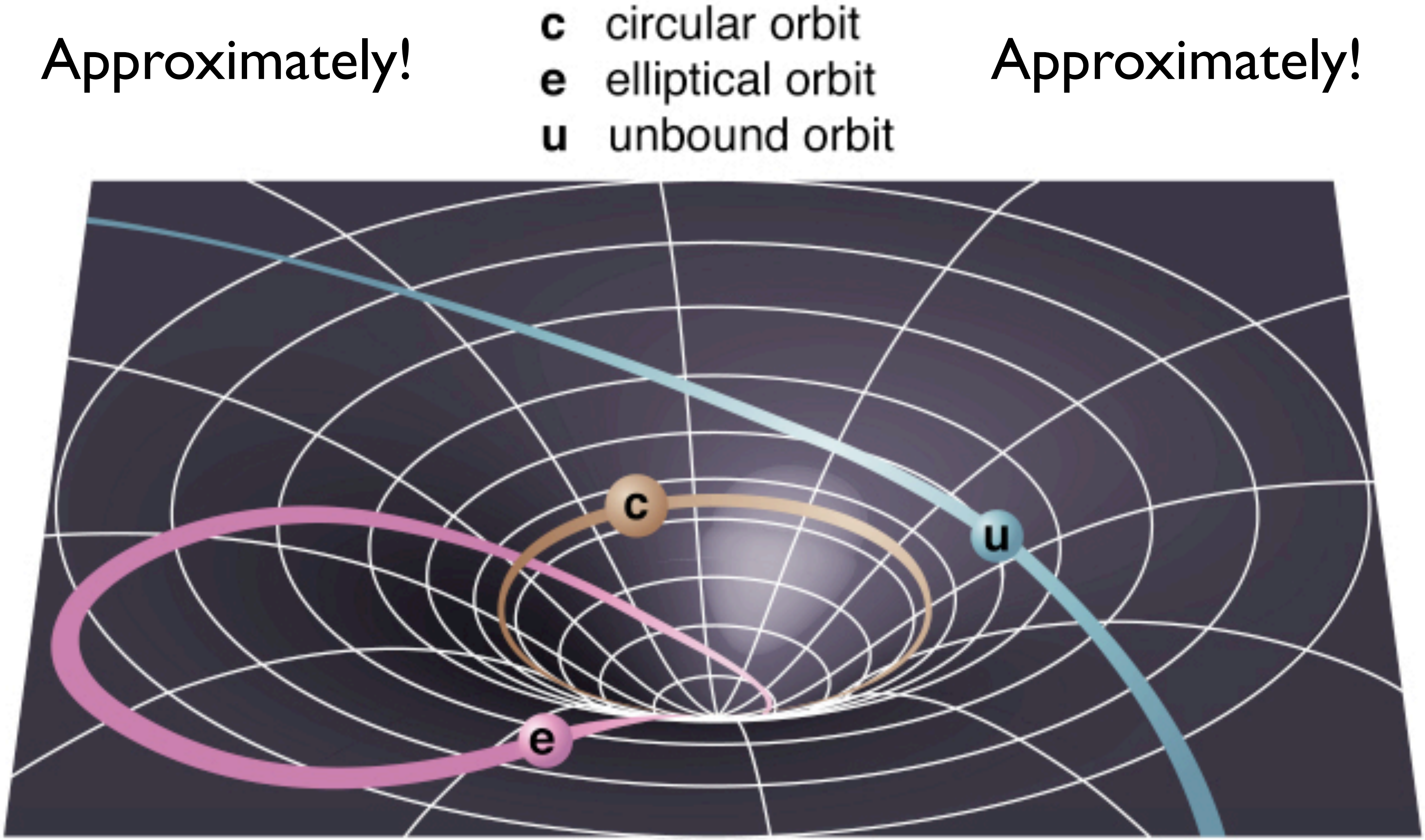
The Einstein equations actually a set of ten coupled nonlinear second-order partial differential equations...

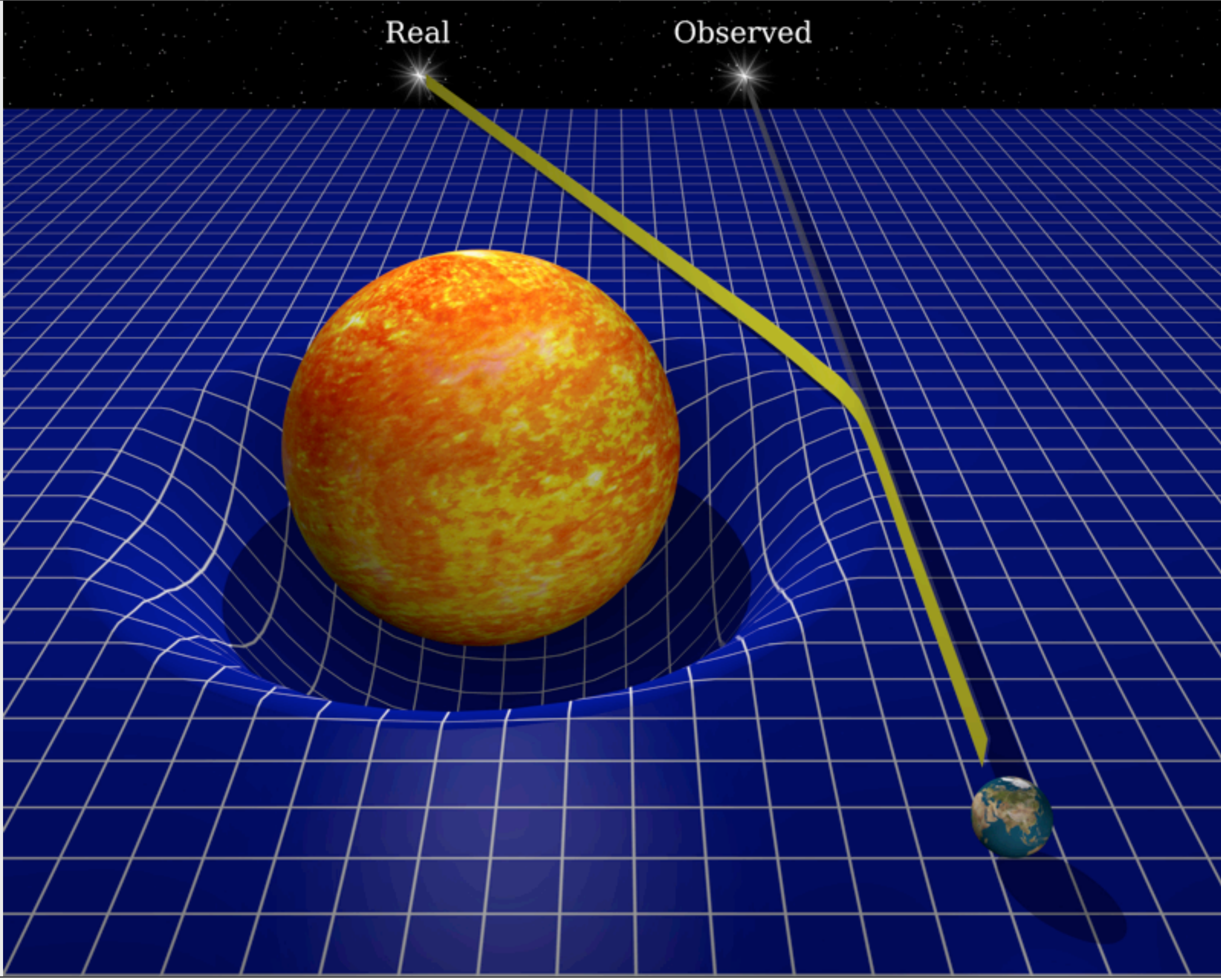
Finding physically relevant exact solutions is often extremely difficult...

Predictions:

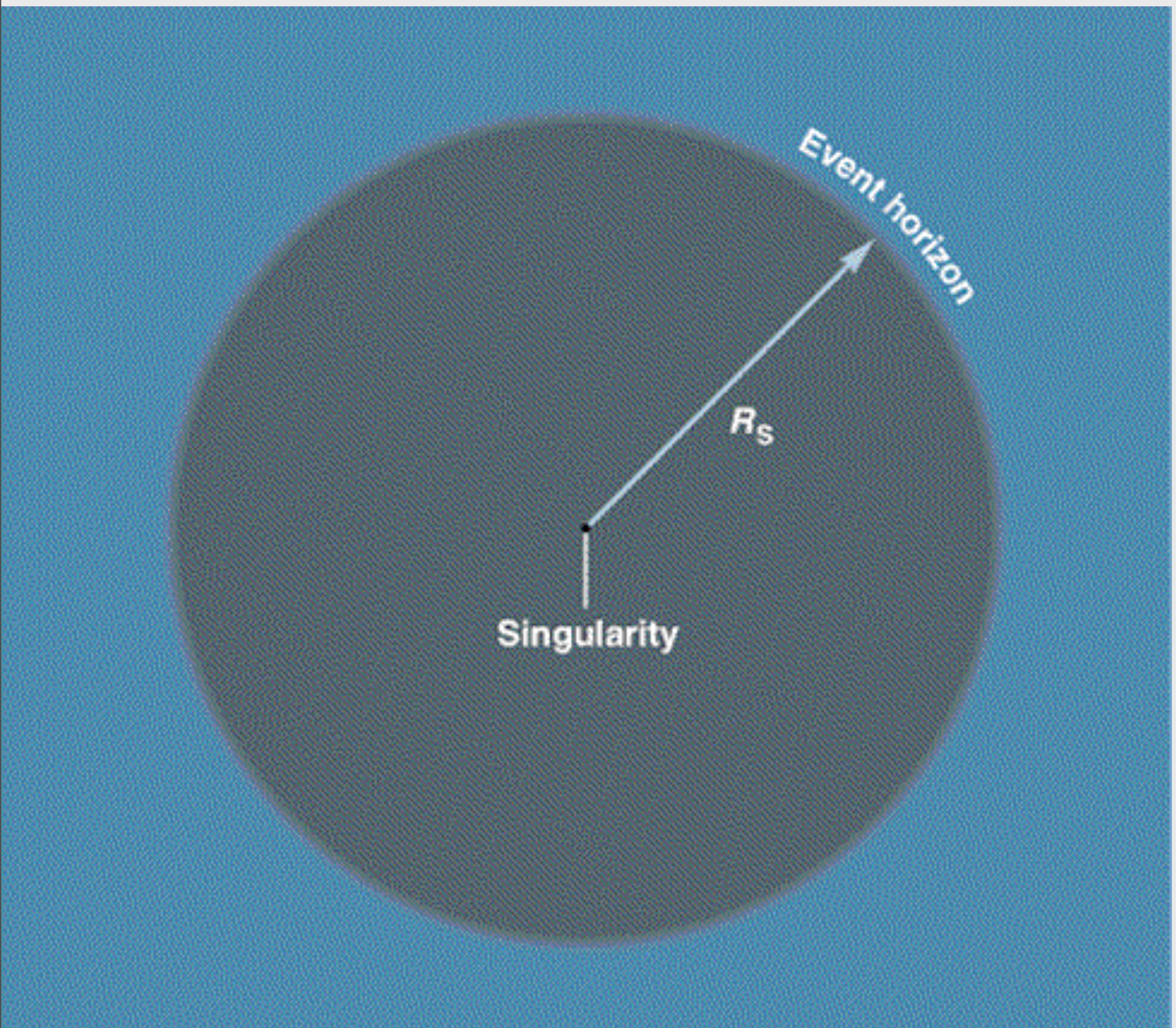
Approximately!

Approximately!

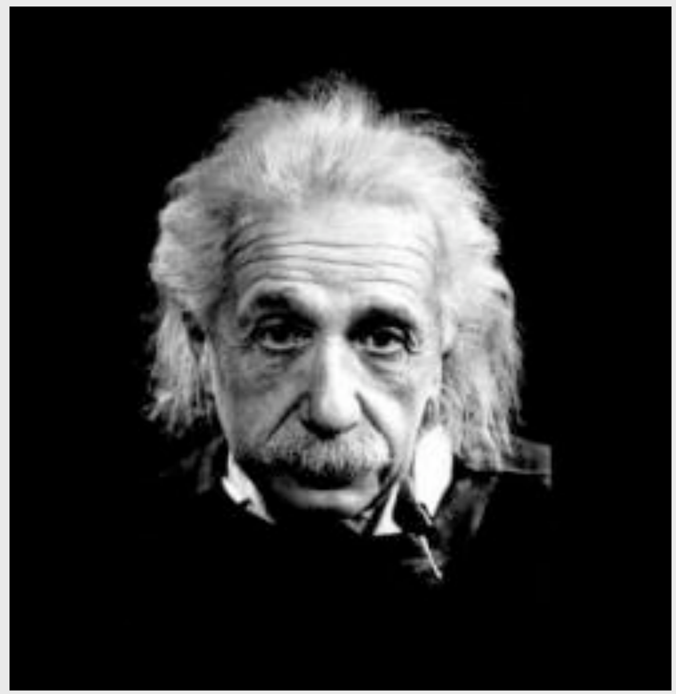


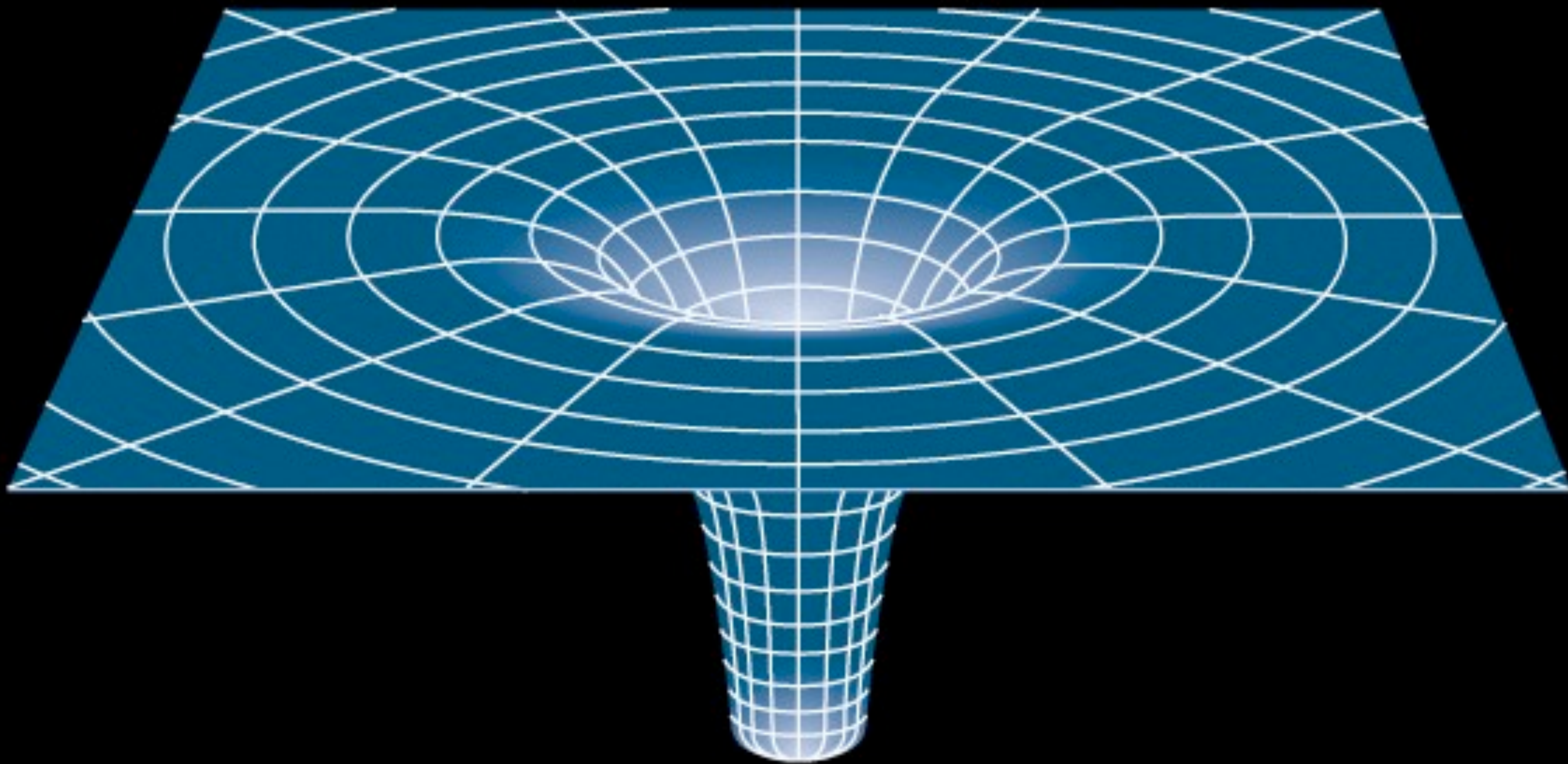

Real
Observed

Black holes:

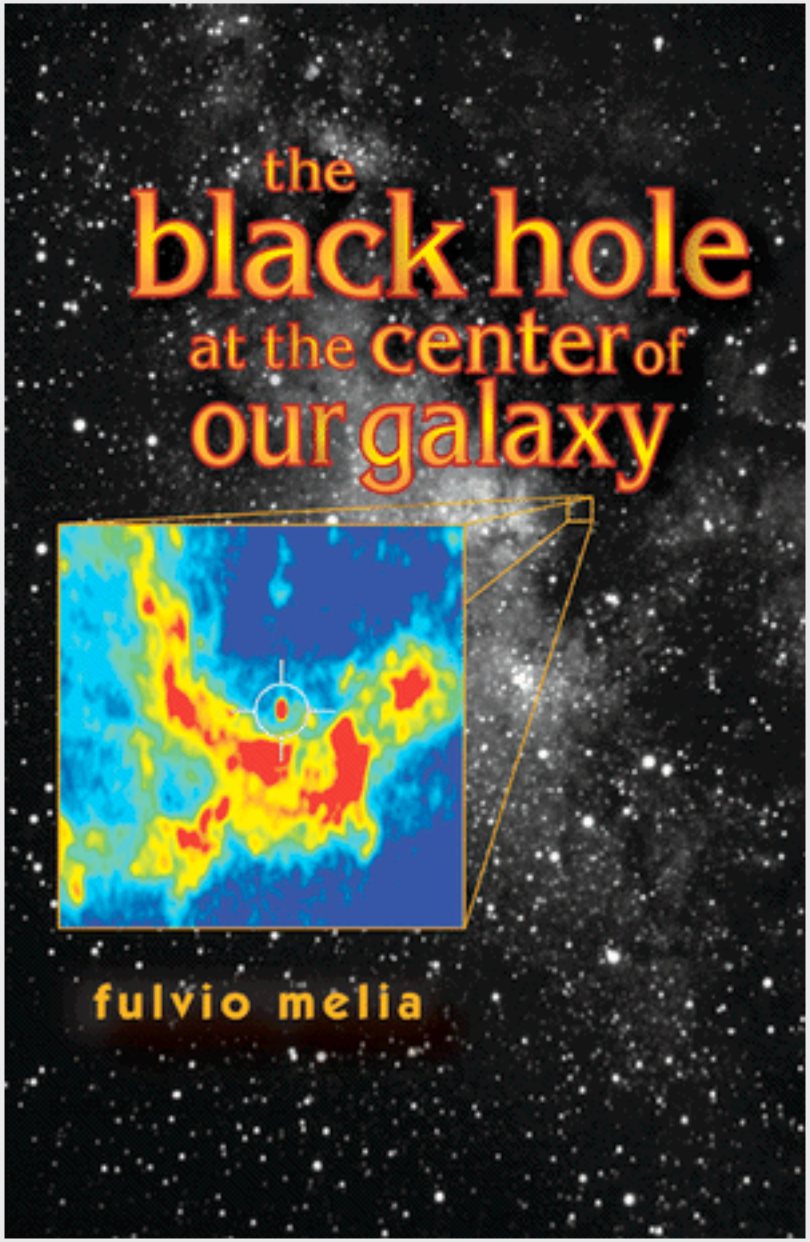


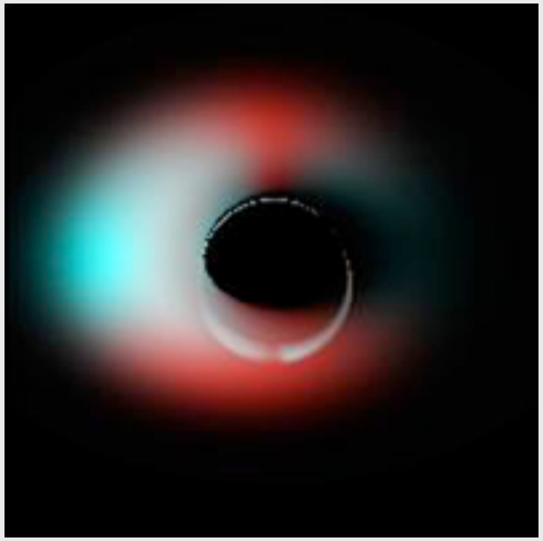
$$R = \frac{2GM}{c^2}$$

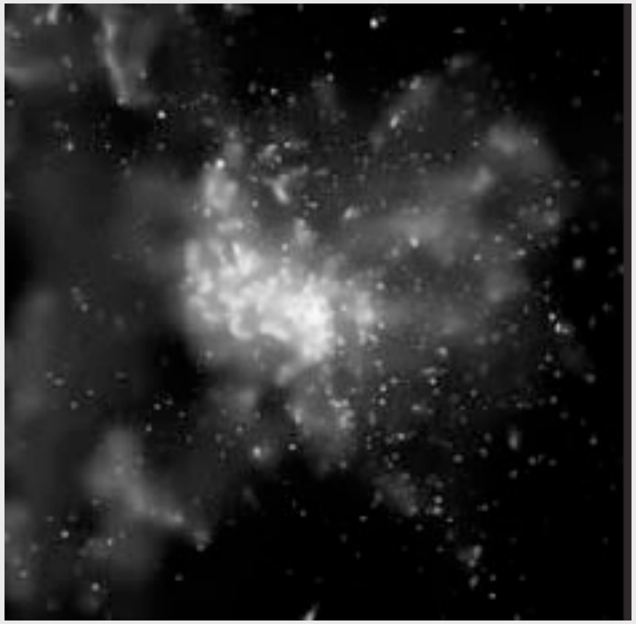


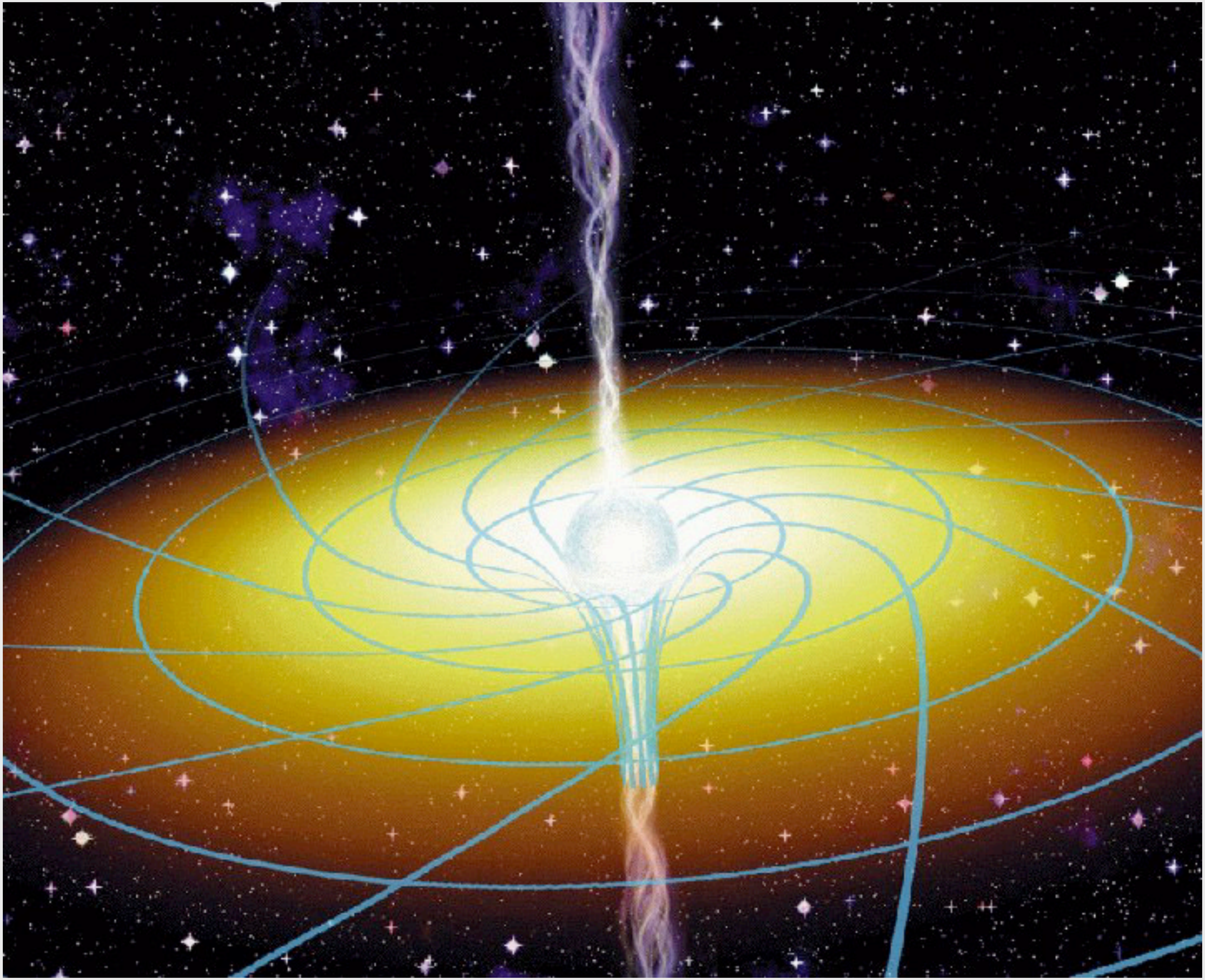


Classically, nothing can escape...

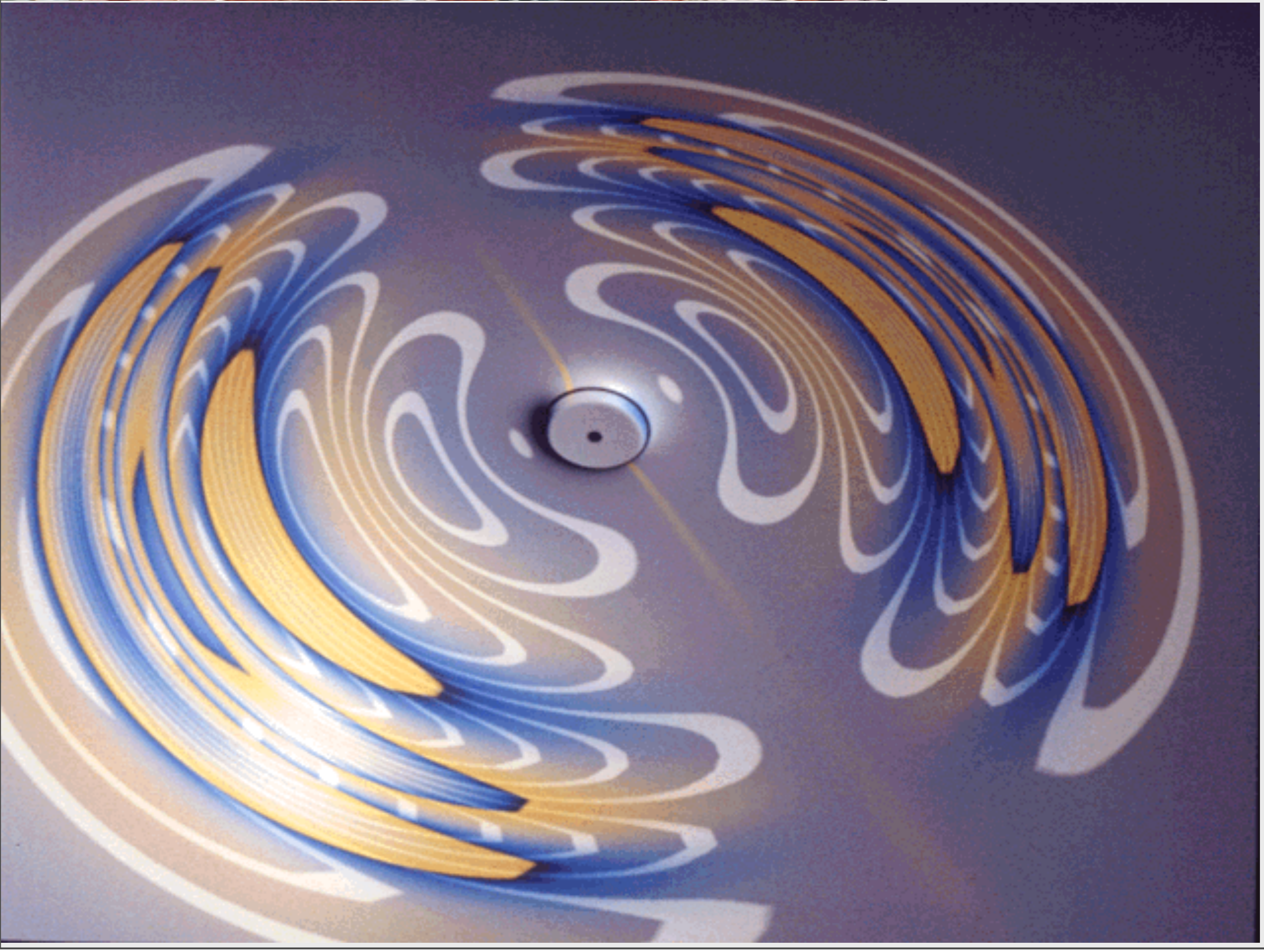

the
black hole
at the center of
our galaxy
fulvio melia







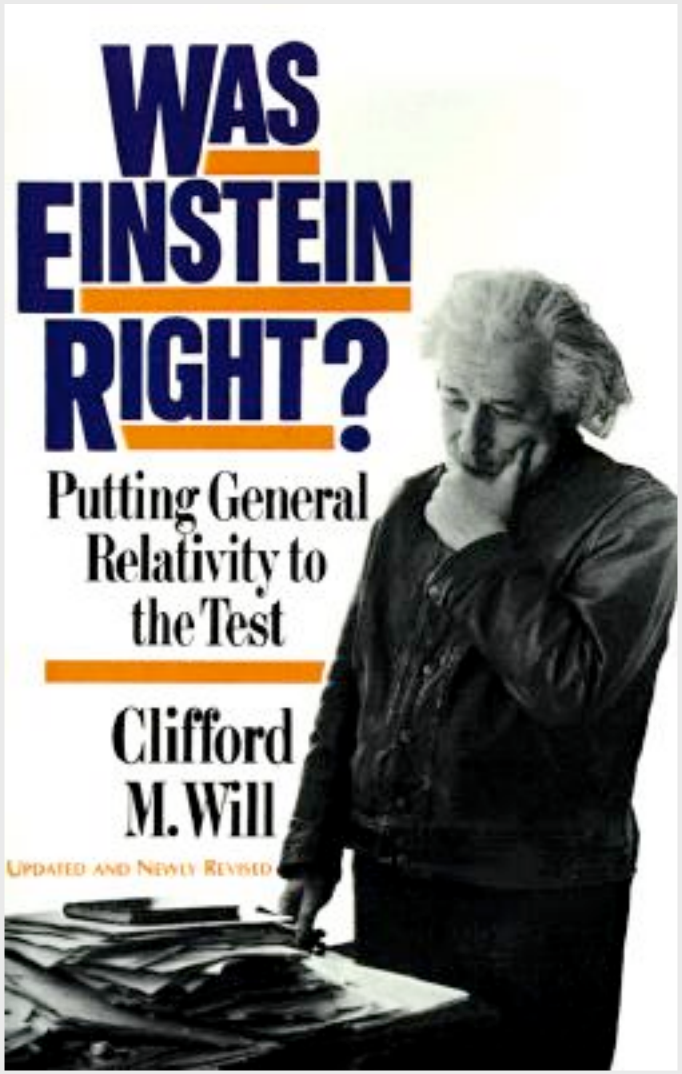
Gravity waves



Ripples
in
space-time

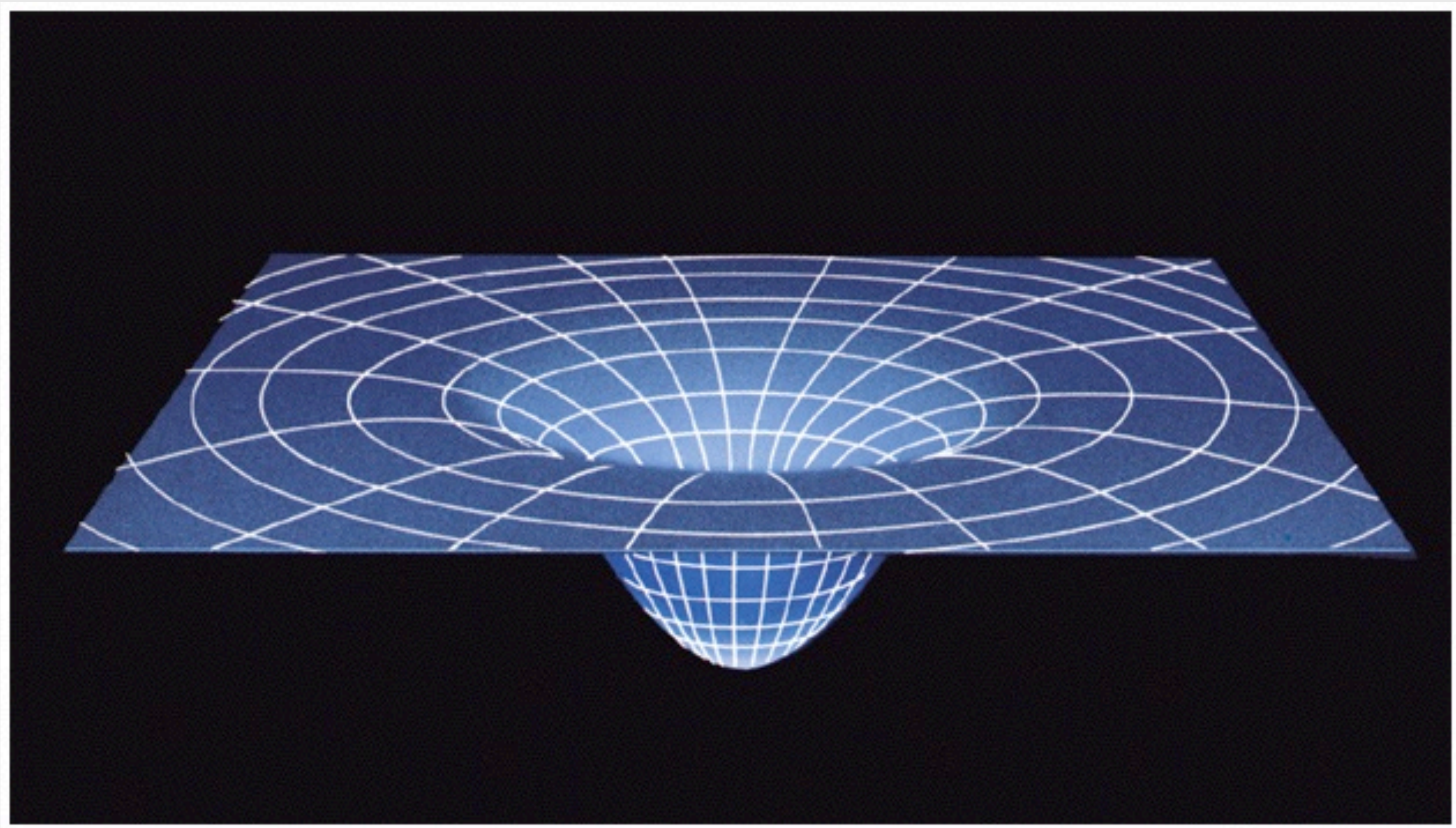
Still
collecting
data...

LIGO



Yes...

Long answer:
Standard
special and general relativity
are completely compatible
with present-day
experiment...



Spacetime curves --- in the manner Einstein predicted.

Now add quantum physics...

Why do we need quantum gravity?

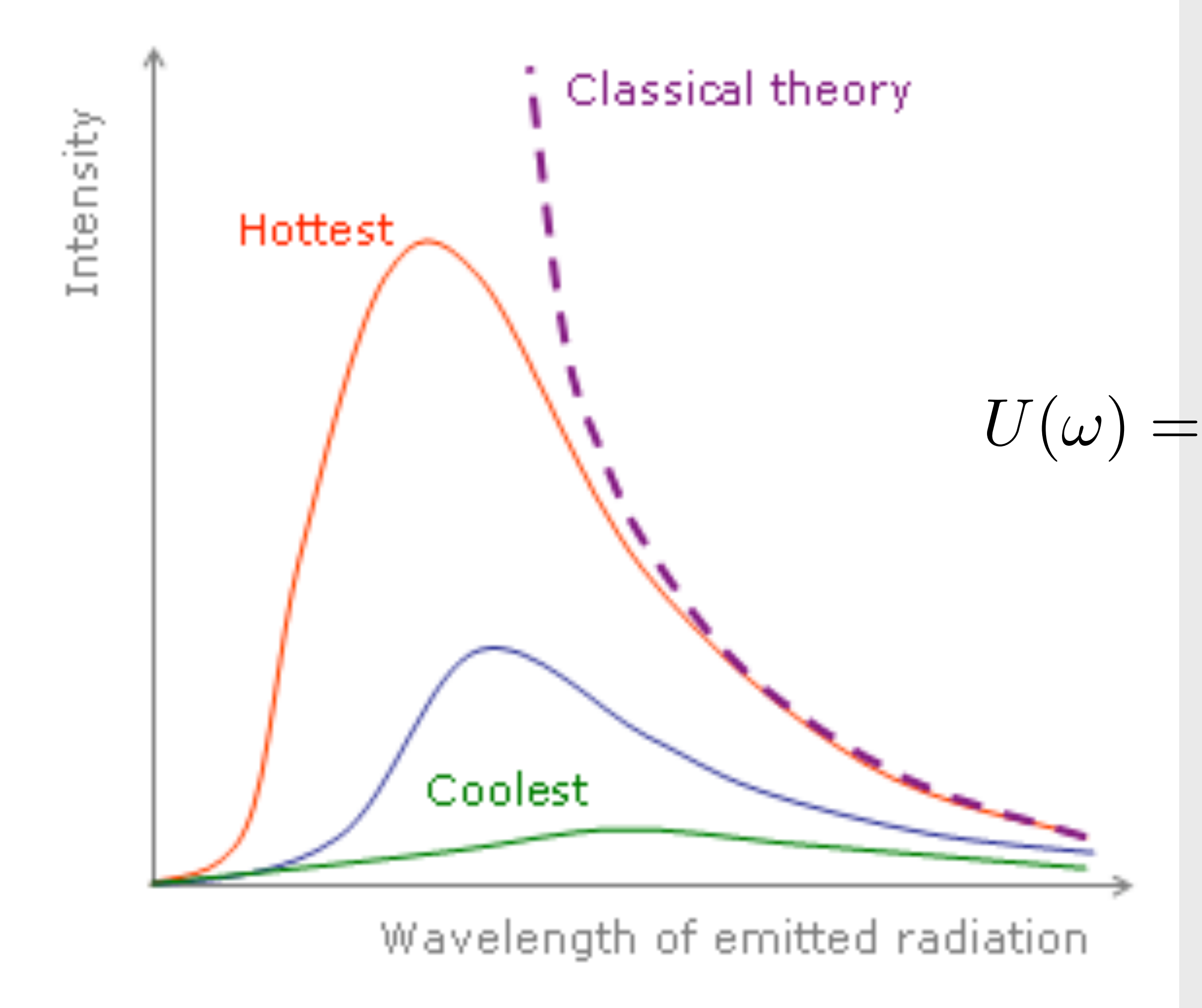
There is this thing called the Planck scale.

First discussed by Max Planck in 1899.

Quantum physics was in its infancy.

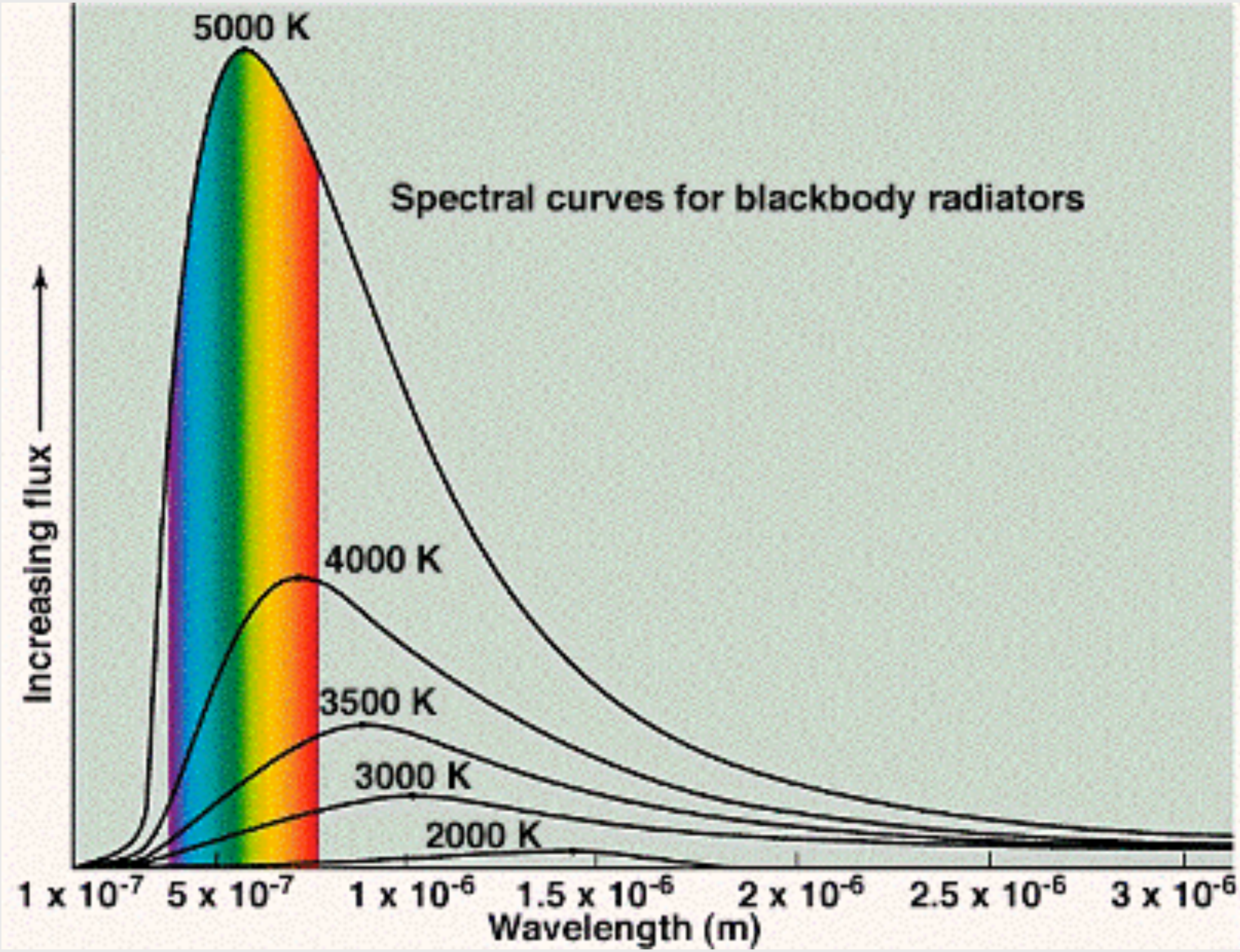
Planck constant — then only an empirical way of parameterizing the unexpected behaviour of black-body radiation.

Planck scale — seemed to be merely an accident of “algebraic numerology”
— you put $\hbar$, c, and G together in various ways and out popped masses, times, and distances.



$$U(\omega) = \frac{8\pi\hbar}{c^3}\left(\frac{\omega}{2\pi}\right)^3 \frac{1}{e^{\hbar\omega/kT} - 1}$$

Black body spectrum


5000 K
Spectral curves for blackbody radiators
Increasing flux
4000 K
3500 K
3000 K
2000 K
1 x 10⁻⁷
5 x 10⁻⁷
1 x 10⁻⁶
1.5 x 10⁻⁶
2 x 10⁻⁶
2.5 x 10⁻⁶
3 x 10⁻⁶
Wavelength (m)

Planck scale

Only after the development of quantum physics (Schrodinger, Heisenberg, 1925) was the significance of the Planck scale as the harbinger of quantum gravity appreciated.

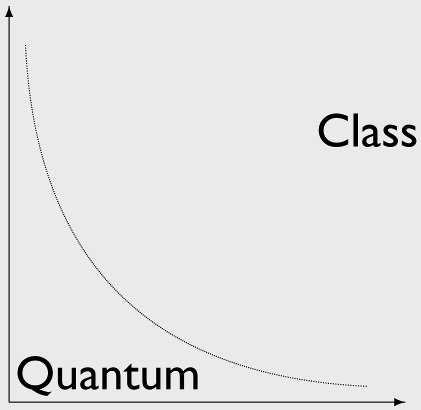
The Planck scale: Compton wavelength

Quantum mechanics tells us that an elementary particle of mass M can be reasonably easily localized within a distance

$$\lambda_{\text{Compton}} = \frac{\hbar}{M\,c}$$

known as the Compton wavelength.

Compton wavelength



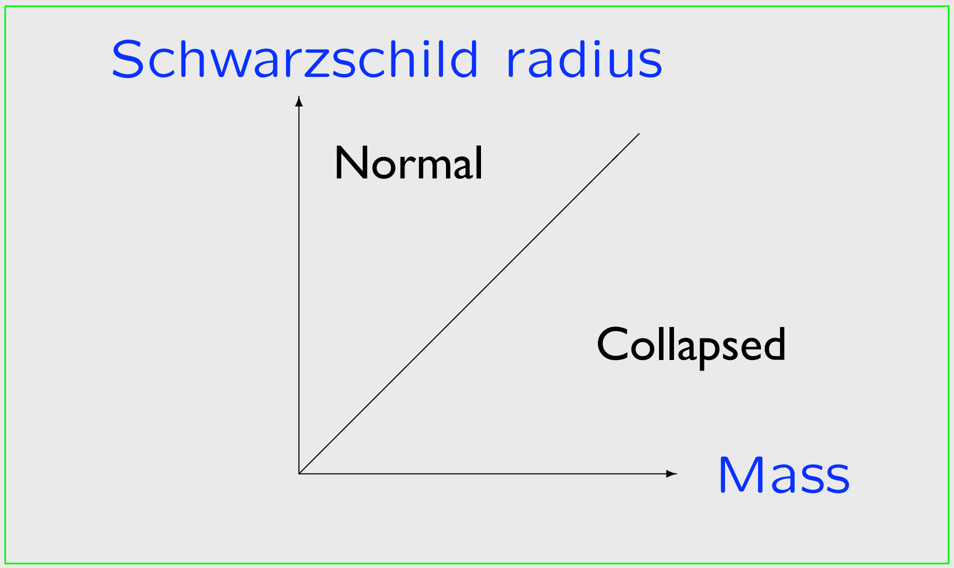
Mass

Compton wavelength as a function of mass.

The Planck scale: Schwarzschild radius

Classical gravity tells us that a particle of mass M will disappear down a black hole if the particle is smaller than its Schwarzschild radius

$$r_{\text{Schwarzschild}} = \frac{2\,G\,M}{c^2}.$$

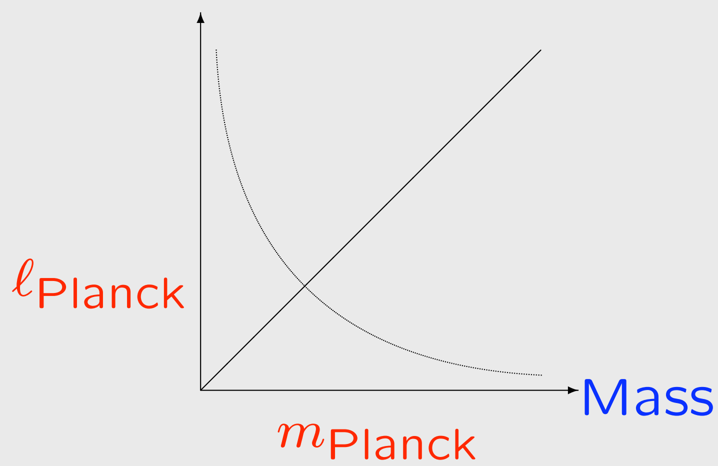


Schwarzschild radius as a function of mass.

The Planck scale: Crossover

Plot the Compton wavelength as a function of mass, and the Schwarzschild radius as a function of mass.

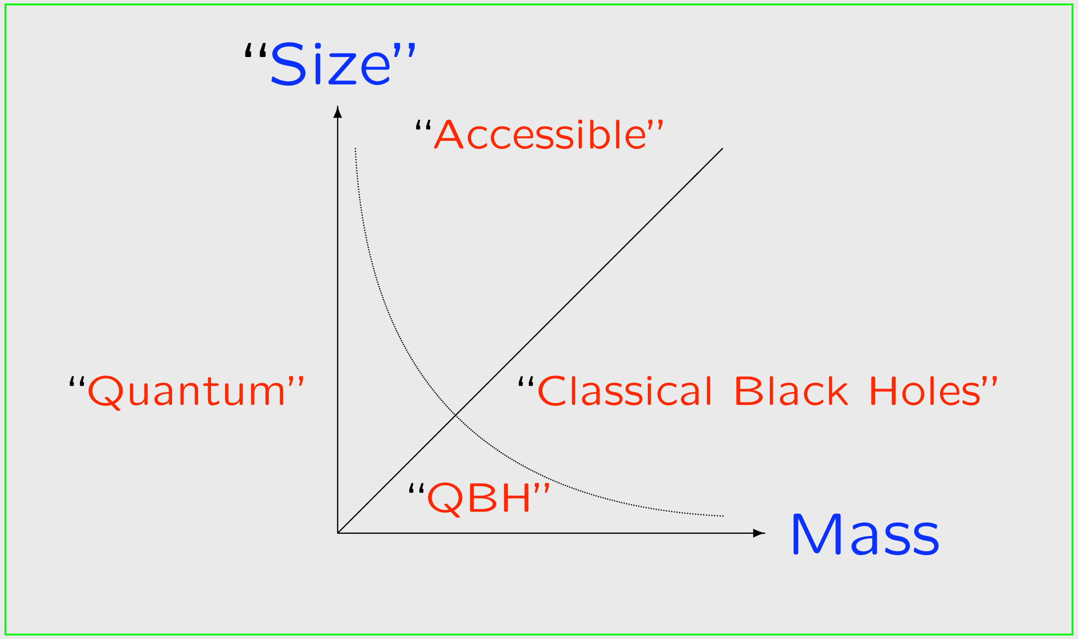
The Planck mass is the place that the two graphs cross.



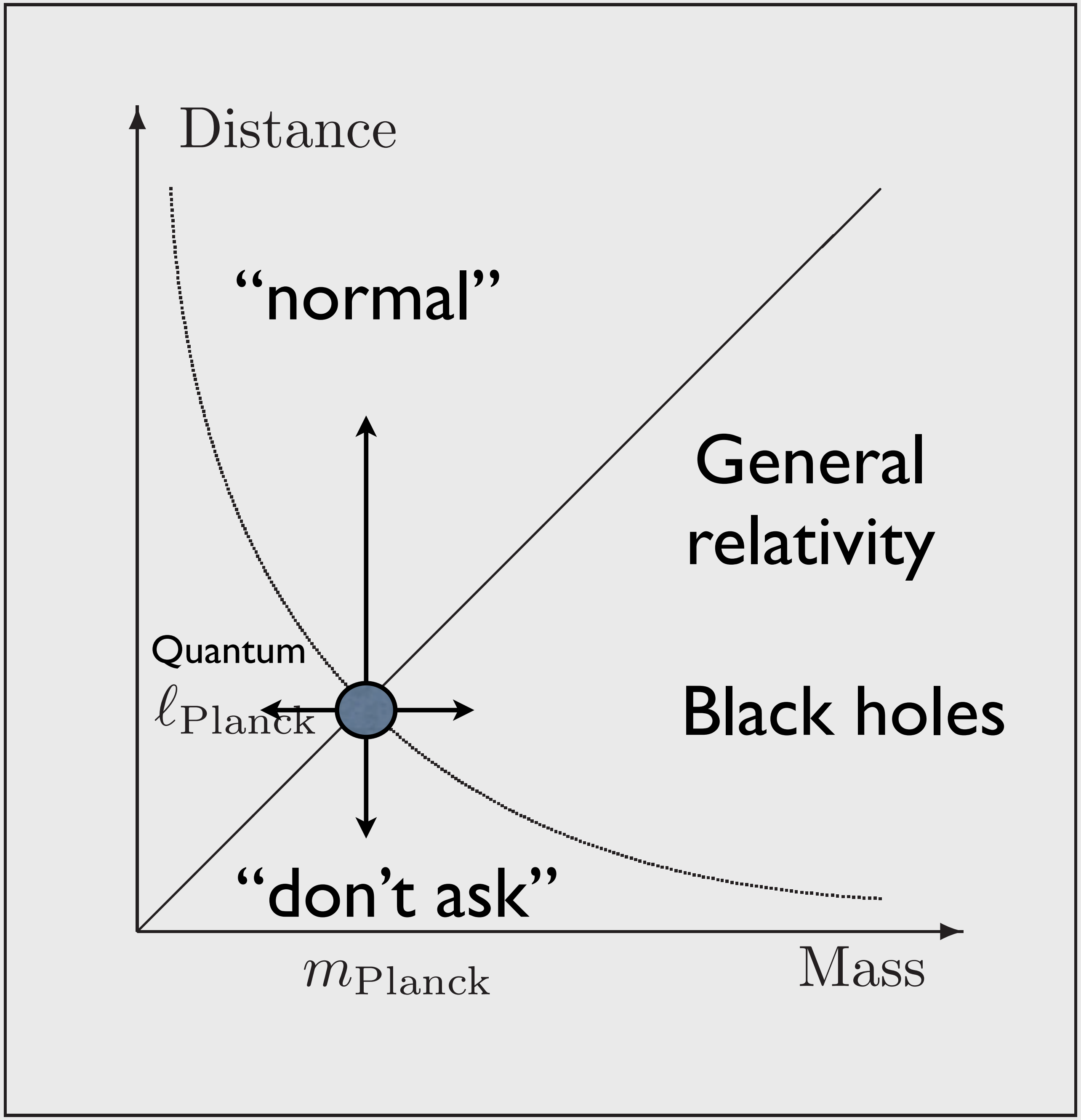
The Planck scale is the crossing point of Compton wavelength and Schwarzschild radius as a function of mass.

Classical–Quantum transition:

- The physically accessible region.



Accessible region for “effective radius” as a function of mass.


Distance
"normal"
General relativity
Quantum
ℓ_{Planck}
Black holes
"don't ask"
m_{Planck}
Mass

Eventually, we will have no choice...

Eventually, we will simply have to address the problem of "quantum gravity"...

What is the quantum theory that reduces to Einstein gravity in some appropriate limit?

Just don't ask too many questions...

Because we are not so sure of the answers...

(at least, not yet...)

String theory, brane theory,
loop quantum gravity,
Lorentzian lattice quantum gravity,
Horava-Lifschitz gravity...

"It is important to keep an open mind; just not so open that your brains fall out"

--- Albert Einstein