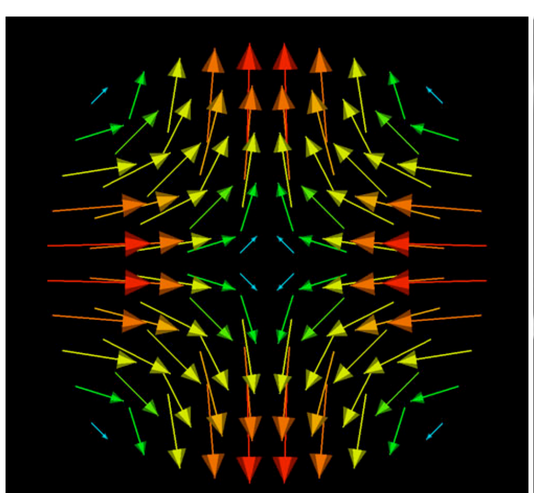
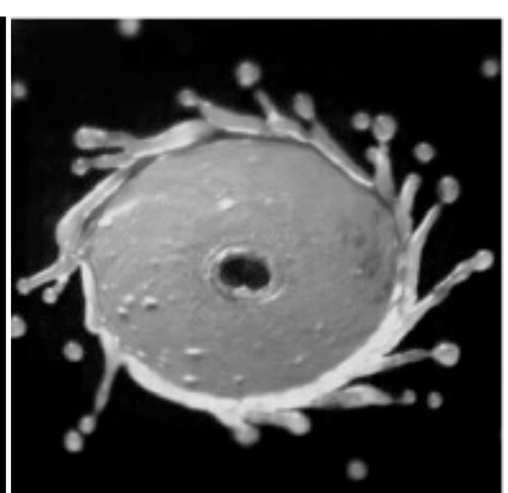
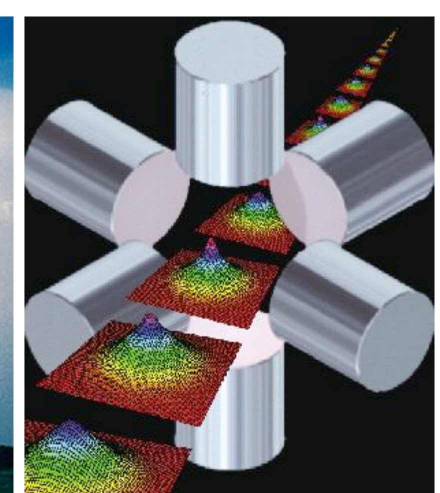
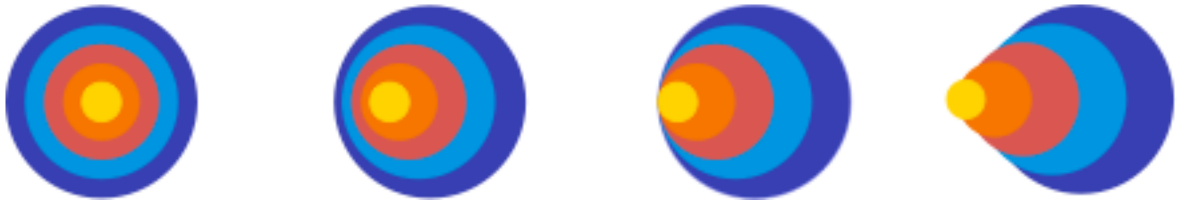
THE UNIVERSITY OF BRITISH COLUMBIA | VANCOUVER

Emergent spacetimes

By Silke Weinfurtner
and collaborators:
Matt Visser, Stefano Liberati, Piyush Jain, Anglea White, Crispin Gardiner and Bill Unruh



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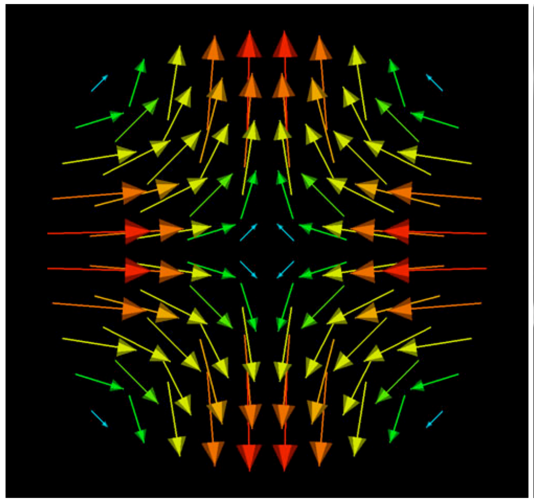
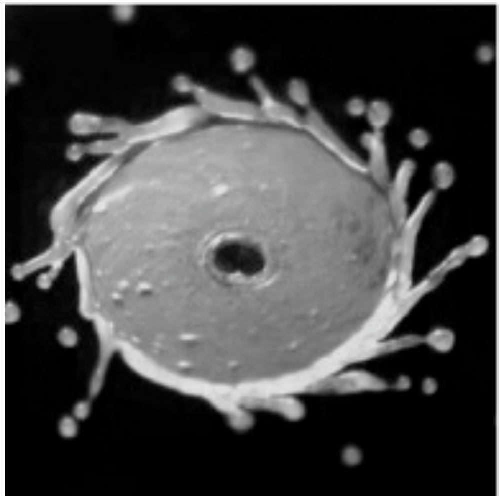
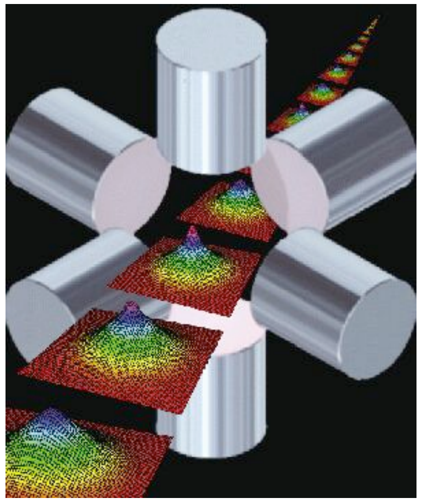
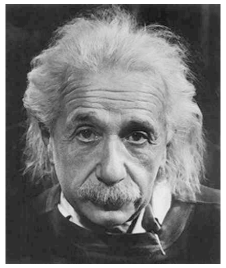
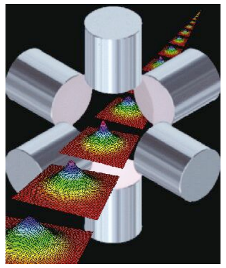






Outlook



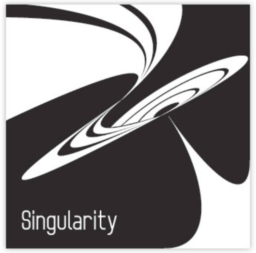
Spacetime geometry and general relativity

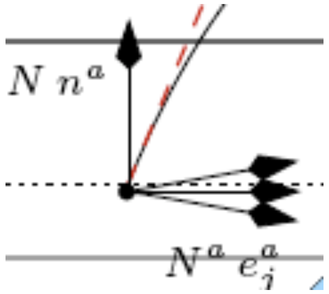


A world full of effective spacetimes

Concept of emergence

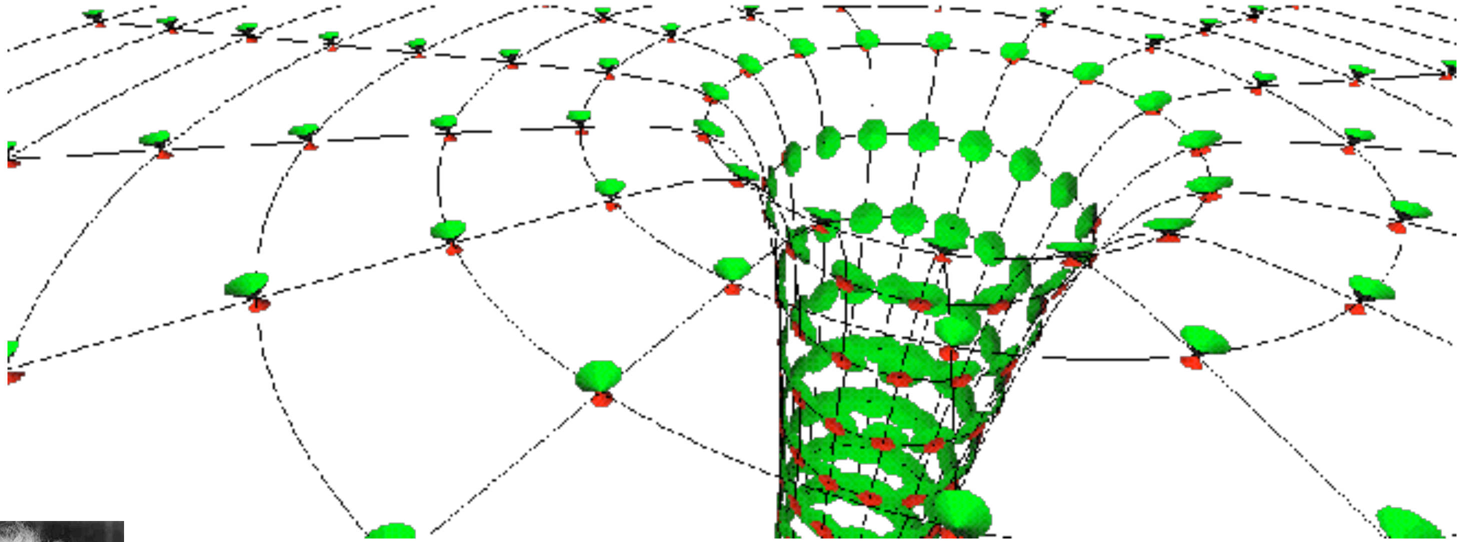
Emergent spacetimes bearing gifts:

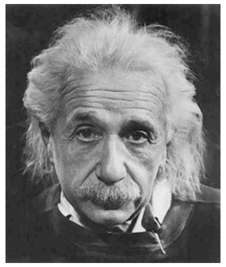




....

From emergent spacetimes to emergent gravity...?





Spacetime geometry and general relativity

Spacetime geometry

Einstein: Gravity a consequence of spacetime geometry!

space (d) + time (1) ➯ spacetime (d+1)

Geometry of spacetime:

$g_{ab} = g_{ab}(t, \mathbf{r})$

$$g_{ab} = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{bmatrix}$$

$n(n+1)/2$ **independent components (without dynamical equations)**

In general relativity *free particles* are *freely falling* particles - no external force but remain under the influence of the spacetime geometry. The kinematical equations of motion for free test particles following geodesics:

$$s_{AB} = \int_A^B ds = \int_A^B \left[-g_{ab}\, dx^a\, dx^b\right]^{1/2}$$

Einstein field equations

$$G_{ab} = 8\pi G_{\mathrm{N}}\, T_{ab}$$

General relativity also identifies (in a coordinate covariant manner) density and flux of energy and momentum in the n dimensional spacetime as the source of the gravitational field g_{ab}:

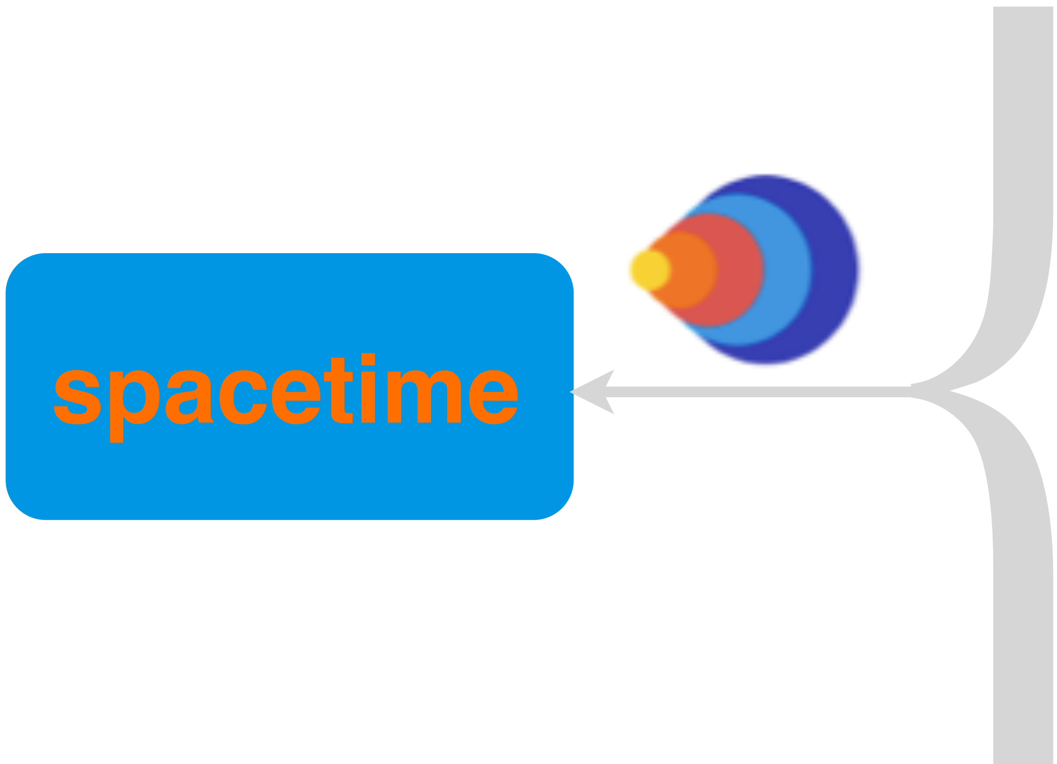
Einstein tensor: $G_{ab} = R_{ab} - \frac{1}{2} R\, g_{ab}$

Stress-energy tensor: $T^{ab} = \left[\begin{array}{c|c} \text{energy density} & \text{energy fluxes} \\ \hline \text{momentum densities} & \text{stress tensor} \end{array}\right]$

Spacetime and gravity

spacetime

Spacetime and gravity



Broad class of systems with ***completely*** different dynamics:

electromagnetic waveguide, fluids, ultra-cold gas of Bosons and Fermions;

Spacetime and gravity

Einstein dynamics:

$$G_{ab} = 8\pi G_{\mathrm{N}}\, T_{ab}$$

→ **spacetime** ←

Broad class of systems with ***completely*** different dynamics:

electromagnetic waveguide, fluids, ultra-cold gas of Bosons and Fermions;

Spacetime and gravity

Einstein dynamics:

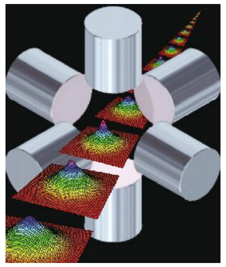
$$G_{ab} = 8\pi G_{\mathrm{N}} T_{ab}$$

spacetime

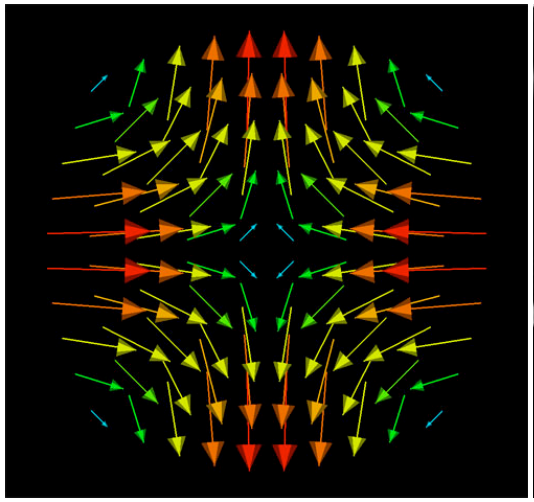
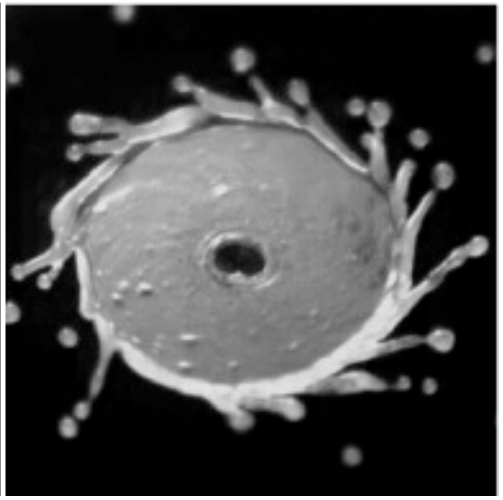
Broad class of systems with ***completely*** different dynamics:

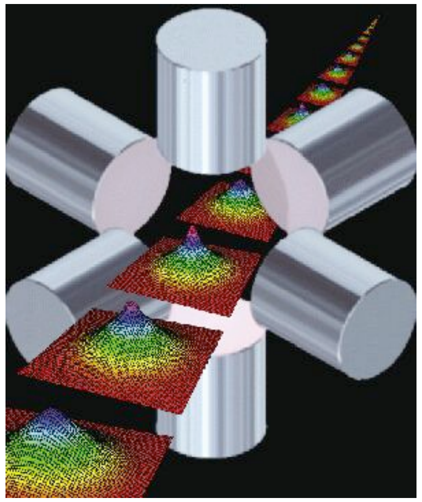
electromagnetic waveguide, fluids, ultra-cold gas of Bosons and Fermions;

Restricted dynamics: Emergent gravity..?



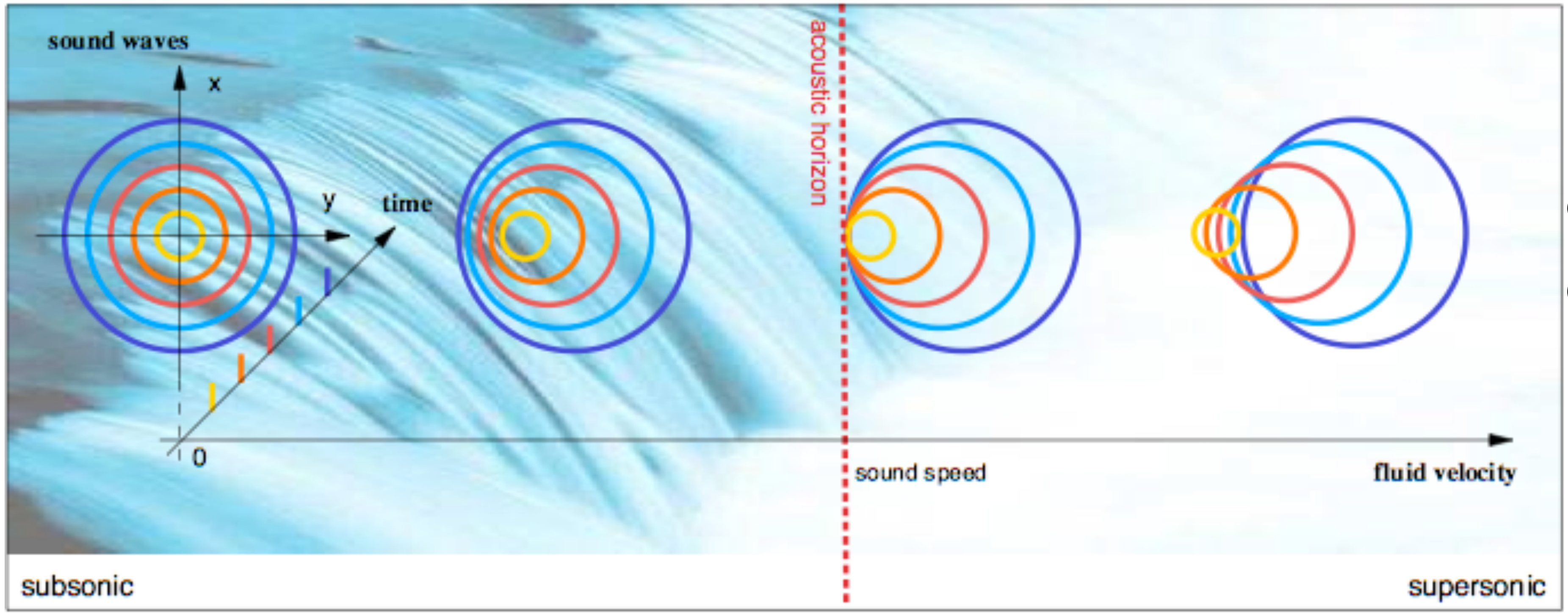
A world full of effective spacetimes




The first analogue model: Schwarzschild bh

A simple example: Sound waves in a fluid flow



Exact analogy to a massless minimally coupled scalar field in an effective curved spacetime:

$$g_{ab} \propto \begin{bmatrix} -(c^2 - v^2) & -v_j \\ -v_i & \delta_{ij} \end{bmatrix}$$

The kinematic equations for small - classical or quantum - perturbations (i.e., sound waves) in barotropic, invicid and irrotational fluid are given by

$$\frac{1}{\sqrt{-g}} \partial_a \left(\sqrt{-g} g^{ab} \partial_b \phi \right) = 0$$

PHYSICAL REVIEW LETTERS

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Experimental Black-Hole Evaporation?

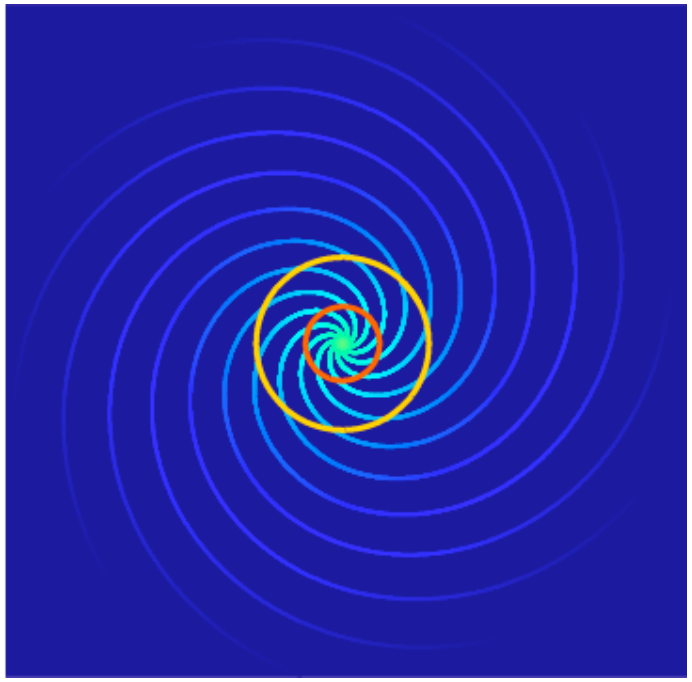
W. G. Unruh

Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada

(Received 8 December 1980)

It is shown that the same arguments which lead to black-hole evaporation also predict that a thermal spectrum of sound waves should be given out from the sonic horizon in transsonic fluid flow.

Acoustic Kerr black hole

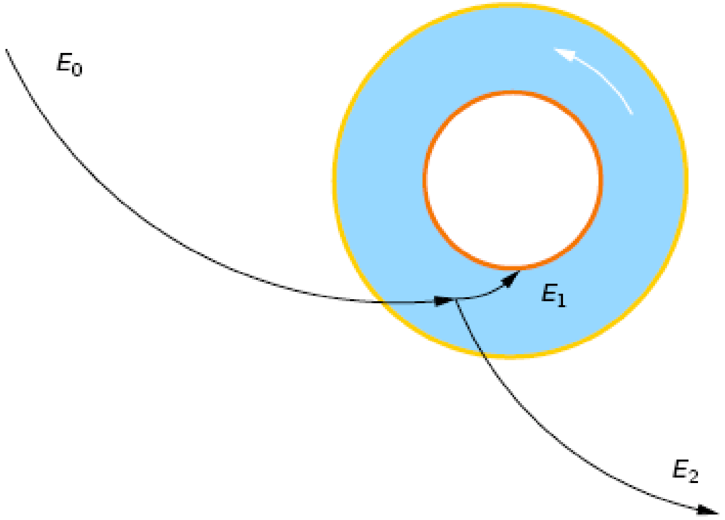


Task:
Can we use a fluid to mimic a Kerr black hole?

Problems:
+ Physical acoustics only for wavelength with:

$$c\,k \gg ||\tilde{\nabla} \times \tilde{v}||$$

+ Only 1 degrees of freedom for acoustic spacetime
+ Conformal flatness of spatial slice in acoustic metric



Relevance: Penrose process; Superradiant Scattering; Hawking radiation

Results: The equatorial slice through a rotating Kerr black hole is formally equivalent to geometry felt by phonons entrained in a rotating fluid vortex.

Natural occurence..?

Natural occurence of acoustic bh

Sound of superradiance?

Idea: *A tornado is a violently rotating column of air, and outside its core it is a perfect example of an irrotational vortex:* $v \propto 1/r$

Problem: *The highest wind speeds recorded do not exceed 150 m/s (about 0.5 Mach), due to different physics in the core of the vortex:* $v \propto r$

Possibility of supersonic wind flows in tornados..?

Shallow water waves - theory

Gravity wave analogs of black holes
Authors: Ralf Schützhold, William G. Unruh
Journal-ref: **Phys.Rev. D66 (2002) 044019**

Propagation of ripples in a basin filled with liquid is governed by

$$ds^2 = \frac{1}{c^2}\left[-(c^2 - (v_B^{//})^2)\,dt^2 - 2\,\vec{v}_B^{//}\cdot d\vec{x}\,dt + d\vec{x}^T\cdot d\vec{x}\right]$$

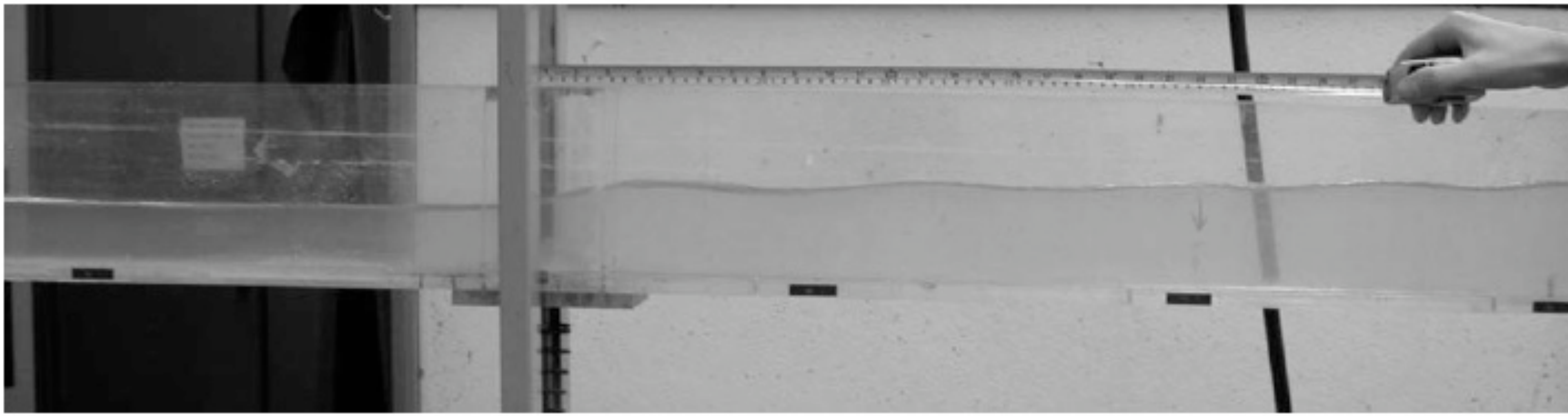
velocity parallel to the basin

Advantage: Extreme ease with which one can adjust the velocity of the surface waves $c^2 = g\,h_B$

Via the depth of the basin!

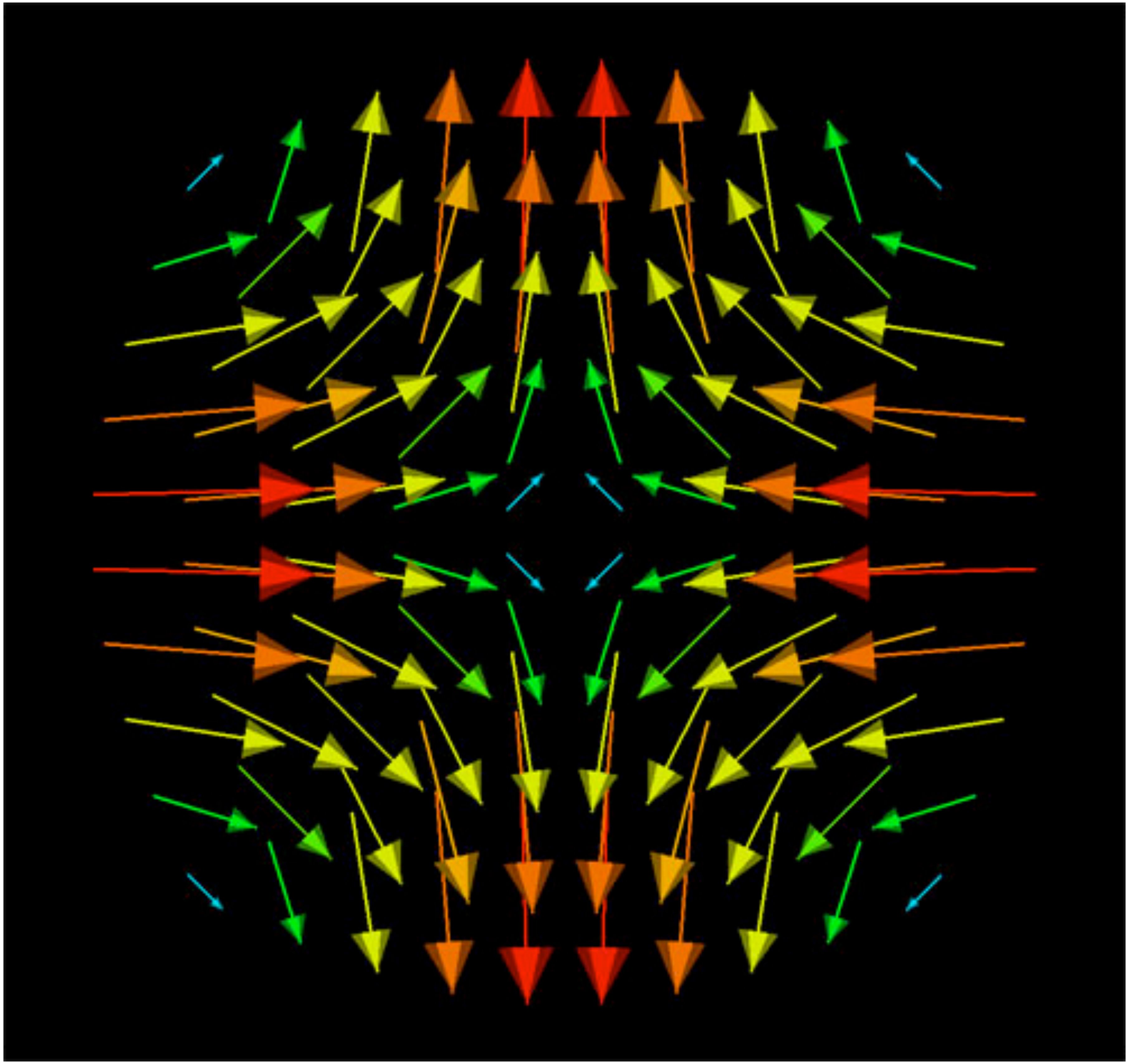
Shallow water waves - experiment

Dumb holes: analogues for black holes
Authors: William G. Unruh
Journal-ref: **Phil. Trans. R. Soc. A 366 (2008) 2905--2913**



flow direction

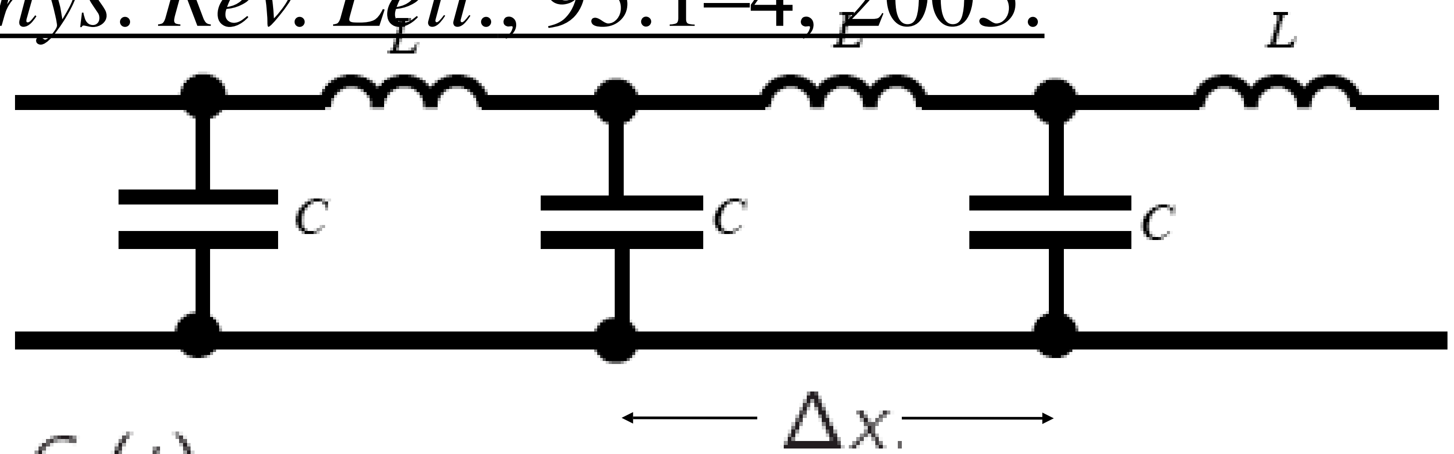
Figure 1. The left-to-right flow is 'supersonic' (faster than the velocity of low-wavenumber gravity waves) to the left of the stanchions and 'subsonic' to the right. The jump in average level at the 'white hole' horizon (where the decreasing velocity of the fluid just equals the velocity of gravity waves) can be regarded as a zero-wavenumber wave impinging on the white hole from the right. The undulating wave (which camps out due to the viscosity of the water) going off to the right has zero-phase velocity, but a non-zero group velocity, carrying energy away from the white hole horizon. Quantum mechanically, this would mean that the radiation from a white hole horizon would not be thermal around zero wavenumber, but rather would be a non-thermal, high-frequency radiation emitted by the white hole horizon.



Electromagnetic wave guide - lattice structure

Manipulating electromagnetic waves in a wave-guide so that they experience an effective curved spacetime in the form of a (2+1) dimensional Painleve-Gullstrand-Lemaitre geometry.

R. Schuetzhold and W. Unruh. *Phys. Rev. Lett.*, 95:1–4, 2005.



Wave-guide consists of a ladder circuit with:

Time-dependent capacitance $C_n(t)$

Coil in each loop has constant inductance $L_n = L = \text{constant}.$

Current in each circuit is given by I_n,

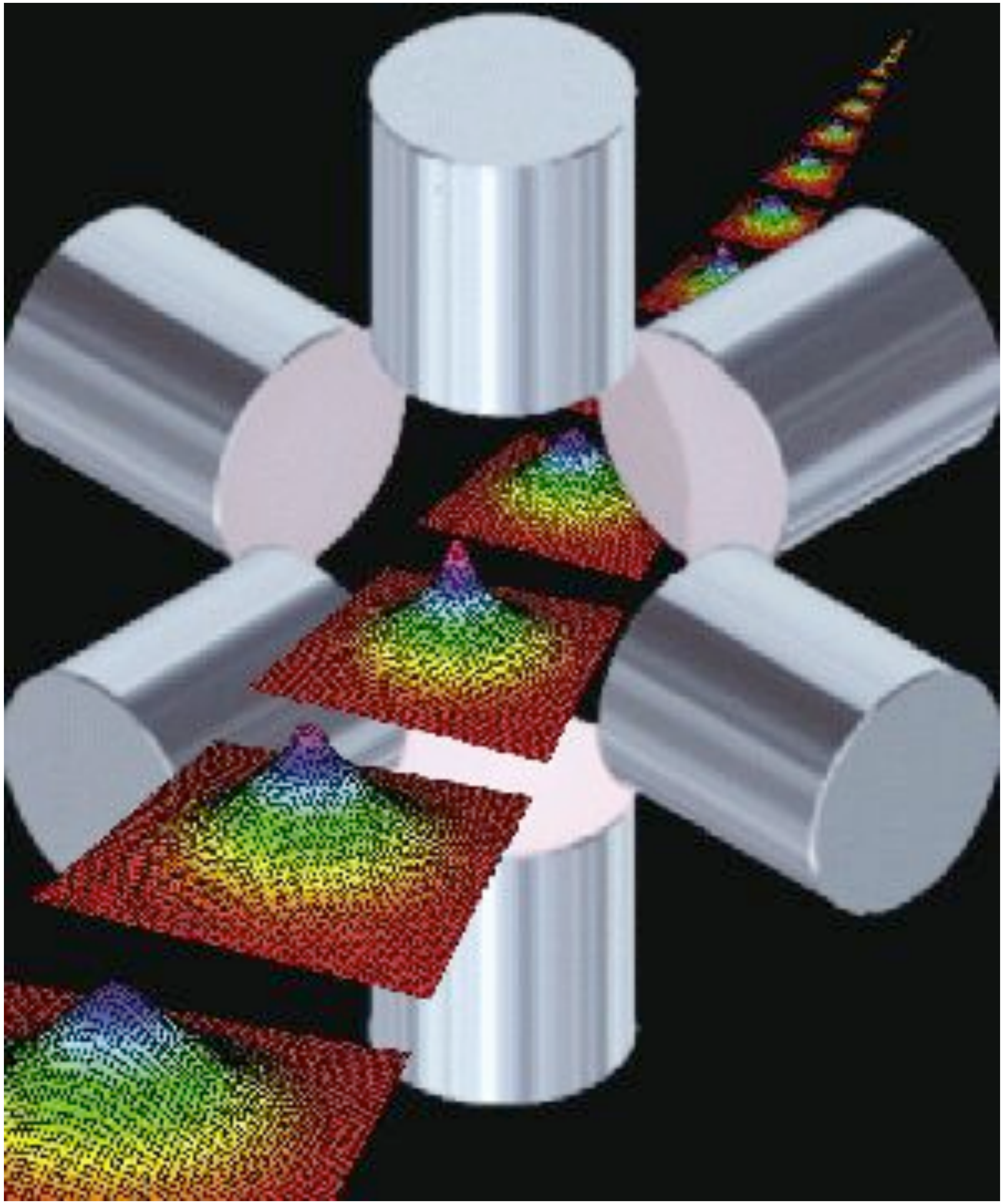
Effective potential A_n , such that $I_n = A_{n+1} - A_n.$

For wave-length $\lambda \gg \Delta x$ the discretness of the x-axis is negligibly small, hence $A_n \to A(x)$:

$$\left[(\partial_t + v\,\partial_x)\,1/c^2\,(\partial_t + v\,\partial_x) - \partial_x^2\right] A(x) = 0$$ where

$$c_n(t) = \Delta x/\sqrt{L\,C_n(t)} \to c(x + v\,t)$$

Bose-Einstein condensate - a quantum model

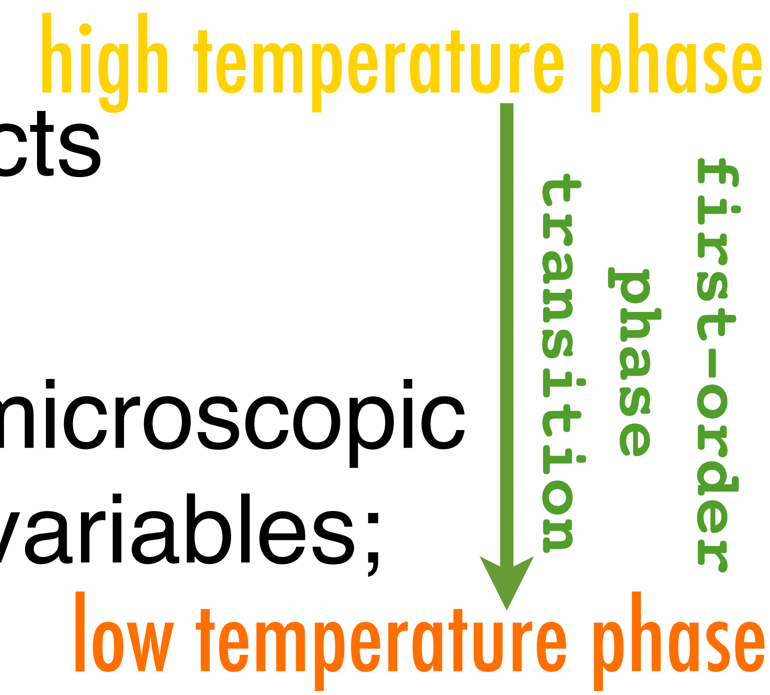


Concept of emergence

The concept of emergence

Emergent spacetimes involve…

- A microscopic system of fundamental objects (e.g. strings, atoms or molecules);
- a dominant mean field regime, where the microscopic degrees of freedom give way to collective variables;



- a geometrical object (e.g. a symmetric tensor dominating the evolution of linearized classical and quantum excitations around the mean field;
- An emergent Lorentz symmetry for the long-distance behavior of the geometrical object;

Example BEC [microscopic degrees of freedom]

Emergent spacetimes from Bose-gas

- A microscopic system of fundamental objects:
  ultra-cold dilute gas of weakly interacting Bosons

Microscopic theory well understood:

$$\hat{H} = \int d\mathbf{x} \left(-\hat{\Psi}^\dagger \frac{\hbar^2}{2m} \nabla^2 \hat{\Psi} + \hat{\Psi}^\dagger V_{\mathbf{ext}} \hat{\Psi} + \frac{U}{2} \hat{\Psi}^\dagger \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi} \right)$$

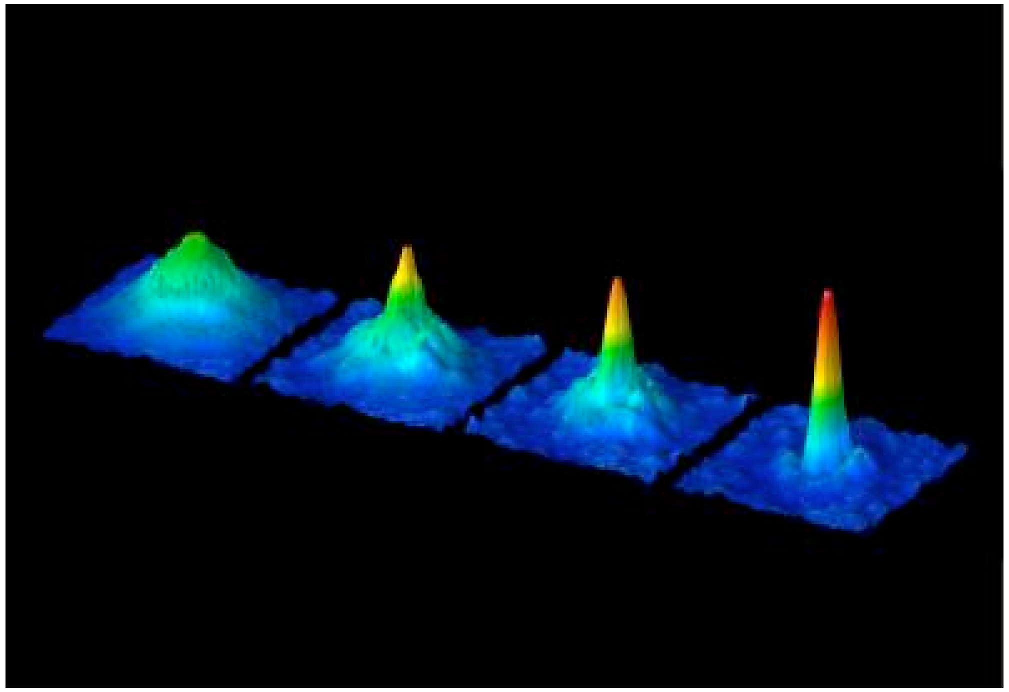
$\mathrm{SO}(2) - \mathrm{symmetry}$ $\hat{\Psi} \rightarrow \hat{\Psi}^* = \hat{\Psi} \exp(i\alpha)$

Example BEC [macroscopic variables]

Emergent spacetimes from Bose-gas

- A dominant mean field regime:

Bose-Einstein condensate



Spontaneous symmetry breaking:

$$\langle \hat{\Psi}(t,\mathbf{x}) \rangle = \psi(t,\mathbf{x}) = \sqrt{n_0(t,\mathbf{x})}\, \exp(i\phi_0(t,\mathbf{x})) \neq 0$$

Example BEC [geometrical object]

Emergent spacetimes from Bose-gas

Small perturbations - linear in density and phase - in the macroscopic mean-field emerging from an ultra-cold weakly interacting gas of bosons are inner observers experiencing an effective spacetime geometry,

$$\frac{1}{\sqrt{|\det(g_{ab})|}}\,\partial_a\left(\sqrt{|\det(g_{ab})|}\;g^{ab}\;\partial_b\hat{\phi}\right)=0$$

where

$$g_{ab}=\left(\frac{c_0}{U/\hbar}\right)^{\frac{2}{d-1}}\begin{bmatrix}-\left(c_0^2-v^2\right) & -v_x & -v_y & -v_z\\ -v_x & 1 & 0 & 0\\ -v_y & 0 & 1 & 0\\ -v_z & 0 & 0 & 1\end{bmatrix};$$

Example BEC [Emergent Lorentz symmetry]

Emergent spacetimes from Bose-gas

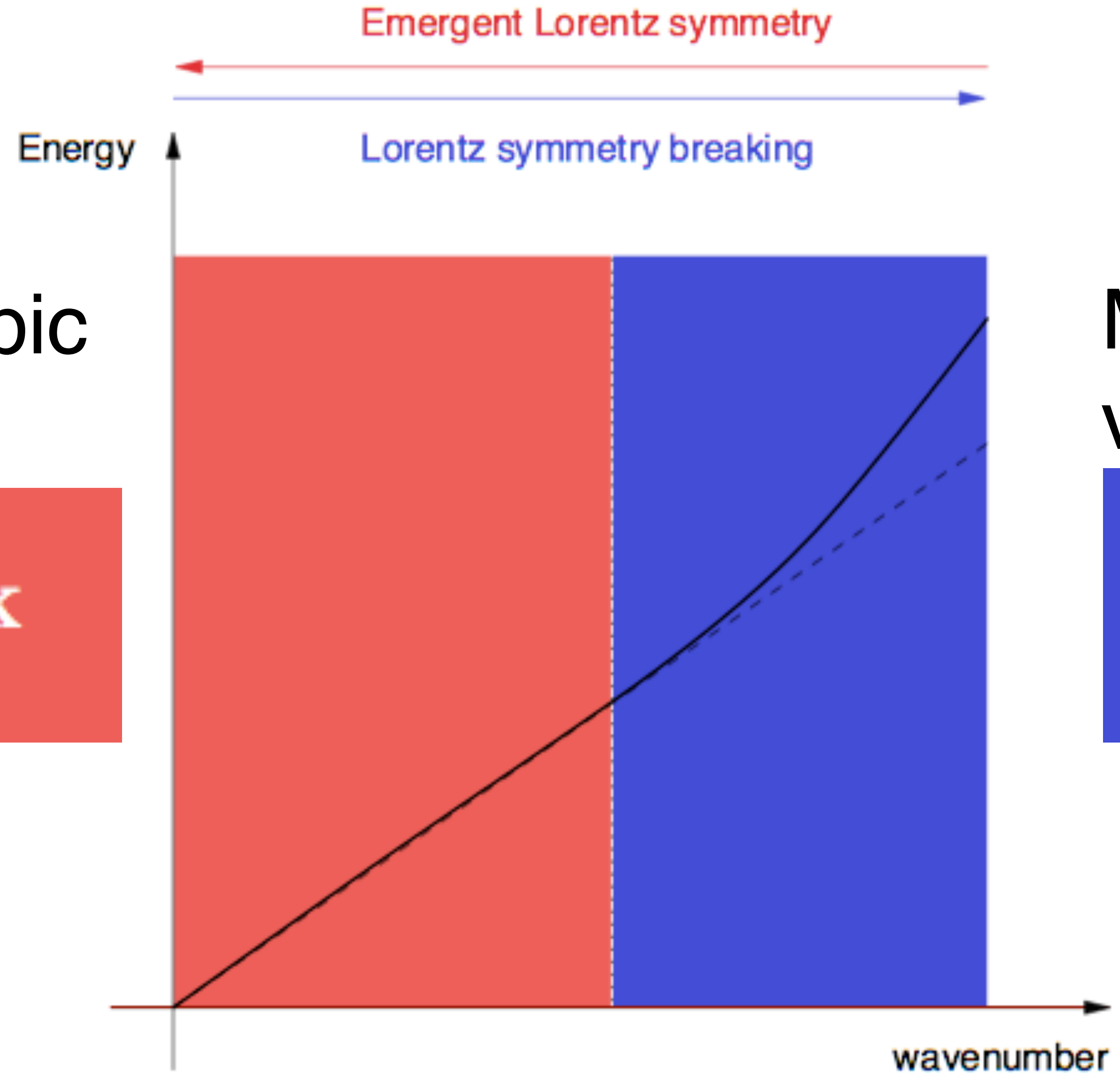
$$\omega_k^2 = c_0^2 k^2 + \left(\frac{\hbar}{2}\right)^2 k^4$$

- An emergent Lorentz symmetry:

Bogoliubov dispersion relation for exciations

Macroscopic variables

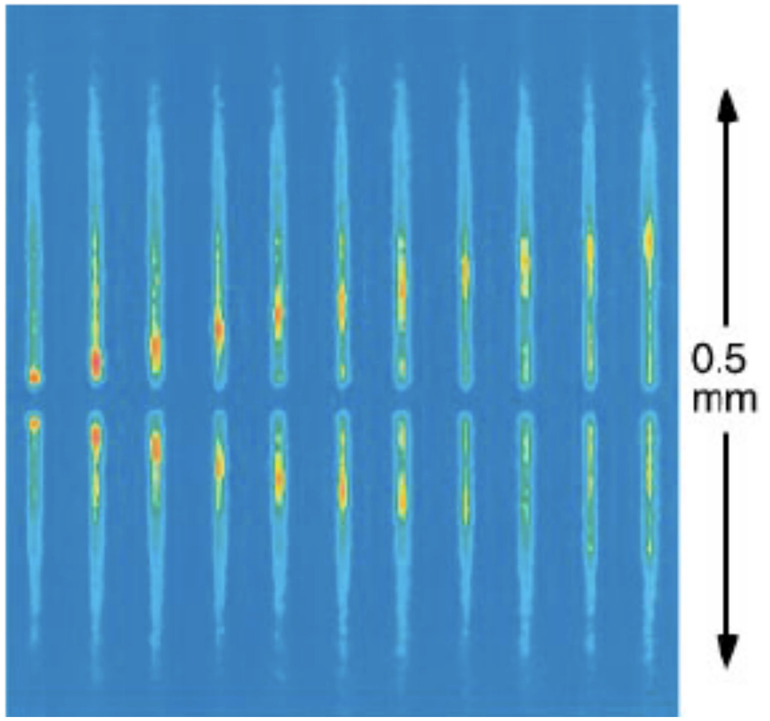
$$\omega_k \approx c_0 k$$



Microscopic variables

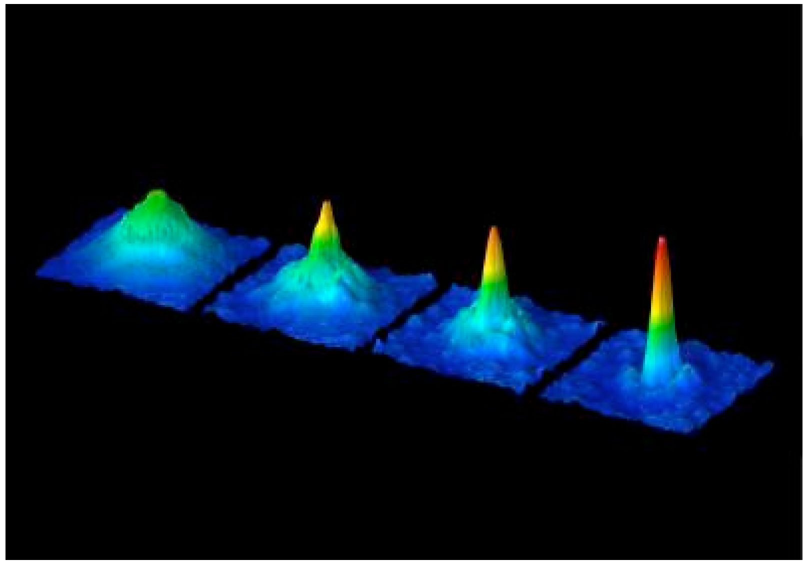
$$\omega_k \approx \frac{\hbar}{2m} k^2$$

On inner and out observer and absolute



Inner observer:
Small excitations in the system experience an effective spacetime geometry represented by the macroscopic mean-field variables!

$$\left[\hat{\theta}(t,\mathbf{x}),\hat{\Pi}_{\hat{\theta}}(t,\mathbf{x}')\right] = i\delta(\mathbf{x}-\mathbf{x}')$$



Outer observer:
Live in the preferred frame - the laboratory frame, such that the condensate parameters are functions of lab-time (absolute time).

$$\left[\hat{n}(t,\mathbf{x}),\hat{\theta}(t,\mathbf{x}')\right] = i\,\delta(\mathbf{x}-\mathbf{x}')$$

Semi-classical quantum geometry

C. Barcelo, S. Liberati, and M. Visser. Analog gravity from field theory normal modes? *Class. Quant. Grav.*, 18:3595–3610, 2001.

Effective curved-spacetime quantum field theory description of the linearization process:

Small perturbations around some background solution $\phi_0(t, \mathbf{x})$

$$\phi(t, \mathbf{x}) = \phi_0(t, \mathbf{x}) + \epsilon\, \phi_1(t, \mathbf{x}) + \frac{\epsilon^2}{2}\, \phi_2(t, \mathbf{x})$$

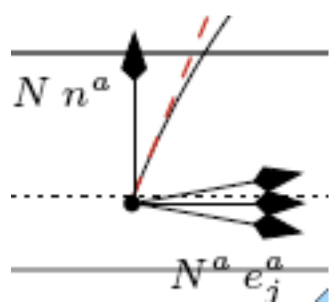
In a generic Lagrangian $\mathcal{L}(\partial_a \phi, \phi)$, depdending only a single Scalar field and its first derivatives yields an effective Spacetime geometry

$$g_{ab}(\phi_0) = \left[-\det\left(\frac{\partial^2 \mathcal{L}}{\partial(\partial_a \phi)\, \partial(\partial_b \phi)} \right) \right]^{\frac{1}{d-1}} \Bigg|_{\phi_0} \left(\frac{\partial^2 \mathcal{L}}{\partial(\partial_a \phi)\, \partial(\partial_b \phi)} \right)^{-1} \Bigg|_{\phi_0}$$

For the classical/ quantum fluctuations. The equation of Motion for small perturbations around the background Are then given by $\left(\Delta_{g(\phi_0)} - V(\phi_0)\right) \phi_1 = 0$

Kinematics versus dynamics!

Emergent spacetimes bearing gifts:



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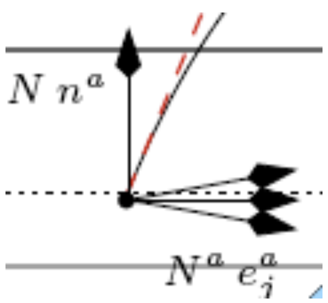


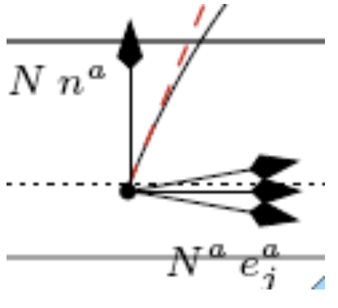
Signature of spacetime

Signature of spacetime - why (-,+,+,+)?

- Signature of spacetime is a certain pattern of Eigenvalues of the metric tensor at each point of the manifold [Loretzian (-,+++) or Riemannian (+,+++)]
- Spacetime foliation into non-intersecting spacelike hypersurfacese (Lapse and Shift)
- Kinematics of signature change (Lapse is a non-dynamical variable)

C. Teitelboim, "The Hamiltonian Structure Of Space-Time", General Relativity and Gravitation 1 (1981) 195–225.

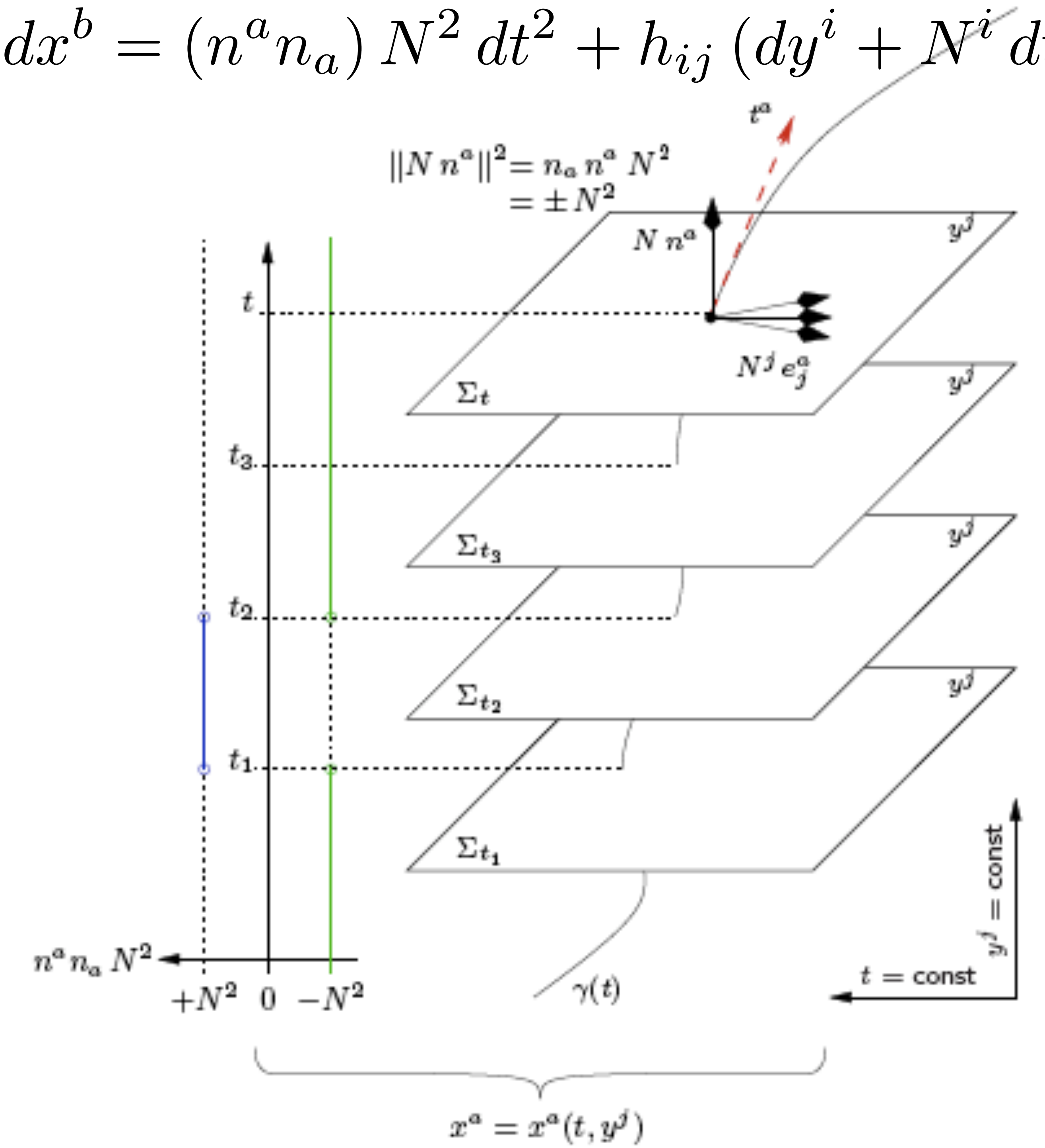
- There is no driving mechanism within GR that drives changes in the signature of the geometry...



Signature of spacetime - what is it really?

➢ Spacetime foliation into non-intersecting spacelike hypersurfacese (Lapse and Shift)

$$ds^2 = g_{ab}\, dx^a dx^b = (n^a n_a)\, N^2\, dt^2 + h_{ij}\,(dy^i + N^i\, dt)\,(dy^j + N^j\, dt)$$



BEC: Interactions ⇢ spacetime signature

$$\vec{v} \to \vec{0}$$

$$U \to U(t)$$

$$c_0^2 \to c(t)^2$$

$$\Rightarrow \quad g_{ab} = \left(\frac{c(t)}{U(t)/\hbar} \right)^{\frac{2}{d-1}} \begin{bmatrix} -c(t)^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_0^2 = \frac{n_0(t, \mathbf{x})\, U(t)}{m}$$

$$U > 0 \qquad \text{repulsive}\,;$$
$$U < 0 \qquad \text{attractive}\,.$$

BEC: Interactions ⇢ spacetime signature

$$\vec{v} \to \vec{0}$$

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$$c_0^2 = \frac{n_0(t, \mathbf{x})\, U(t)}{m}$$

$$U > 0 \quad \text{repulsive};$$
$$U < 0 \quad \text{attractive}.$$

$$g_{ab} \sim \begin{bmatrix} -c(t)^2 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix}$$

Lorentzian signature

$$g_{ab} \sim \begin{bmatrix} +c(t)^2 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix}$$

Riemannian signature

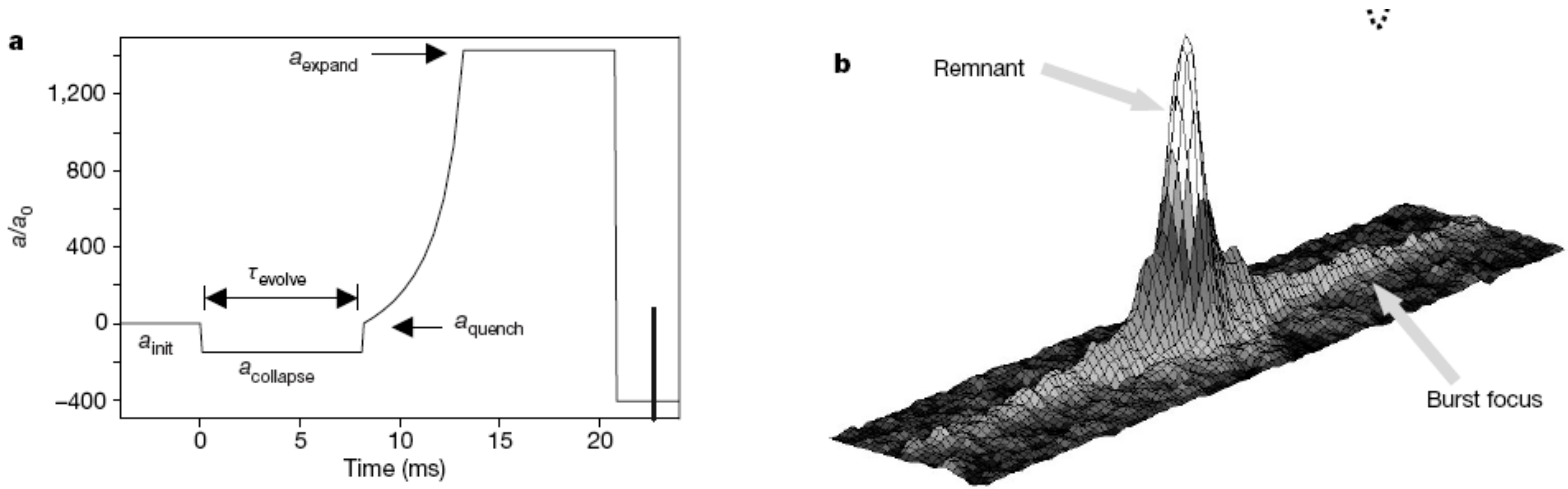
Daily signature change events...

Dynamics of collapsing and exploding Bose–Einstein condensates

NATURE | VOL 412 | 19 JULY 2001

Elizabeth A. Donley*, Neil R. Claussen*, Simon L. Cornish*, Jacob L. Roberts*, Eric A. Cornell*† & Carl E. Wieman*

The bosenova experiment:



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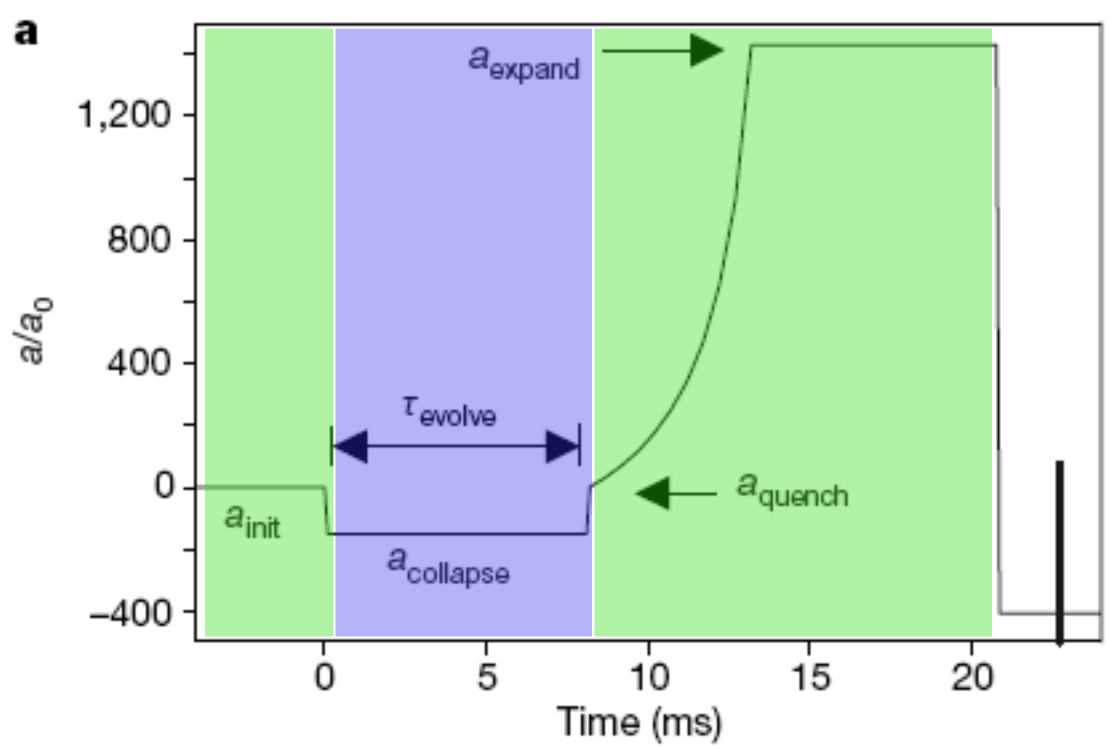
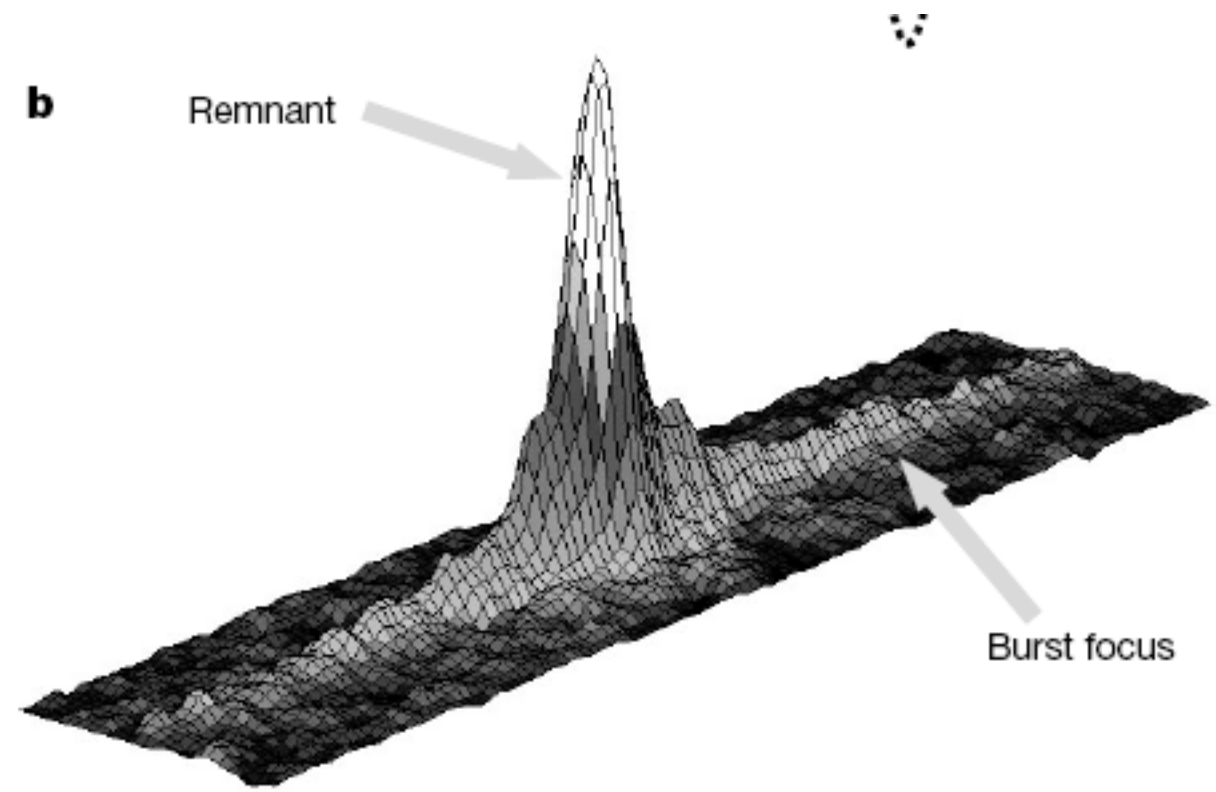
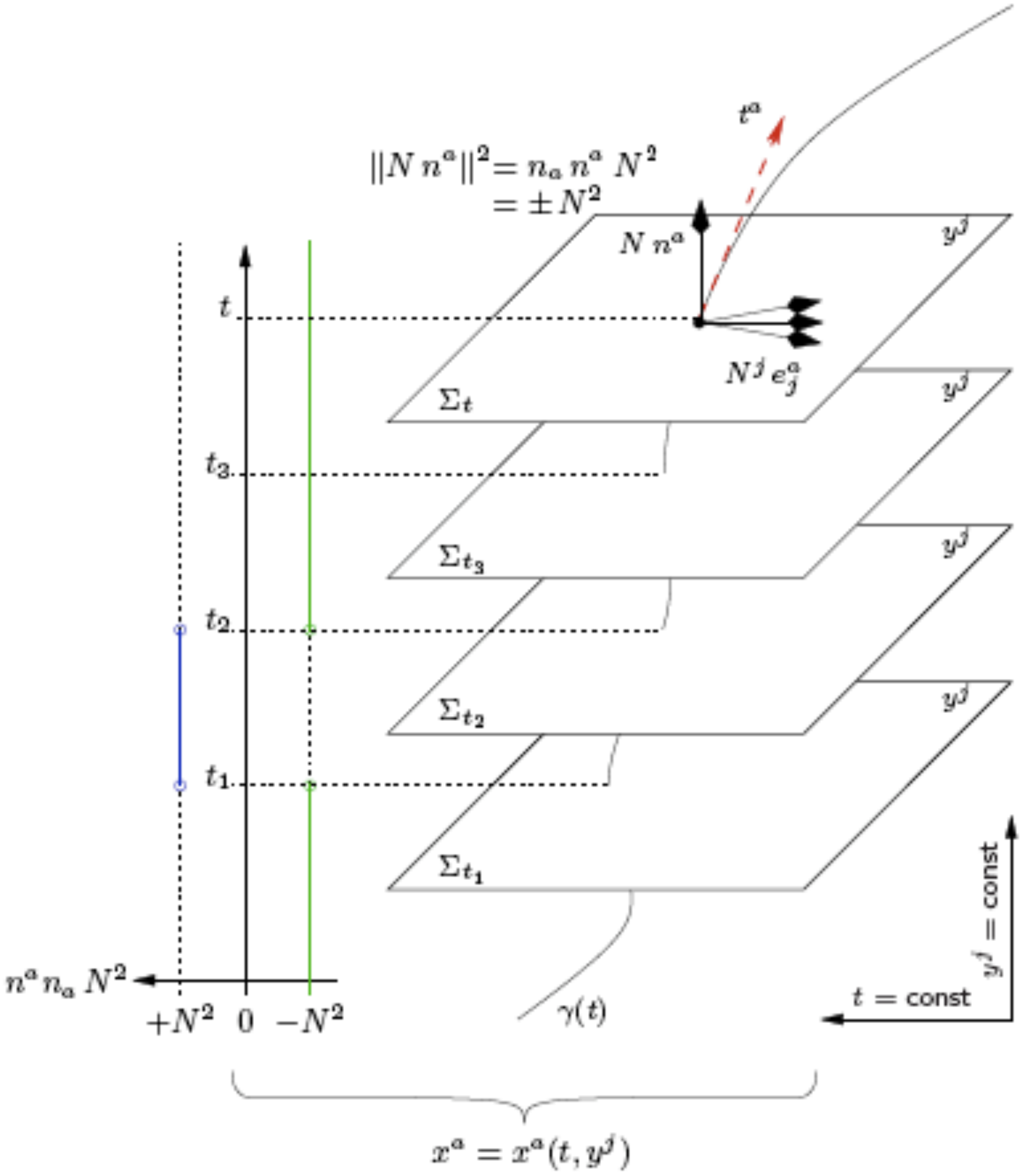




Lorentzian signature 

Riemannian signature 

Quantum field theory on Riemannian manifolds



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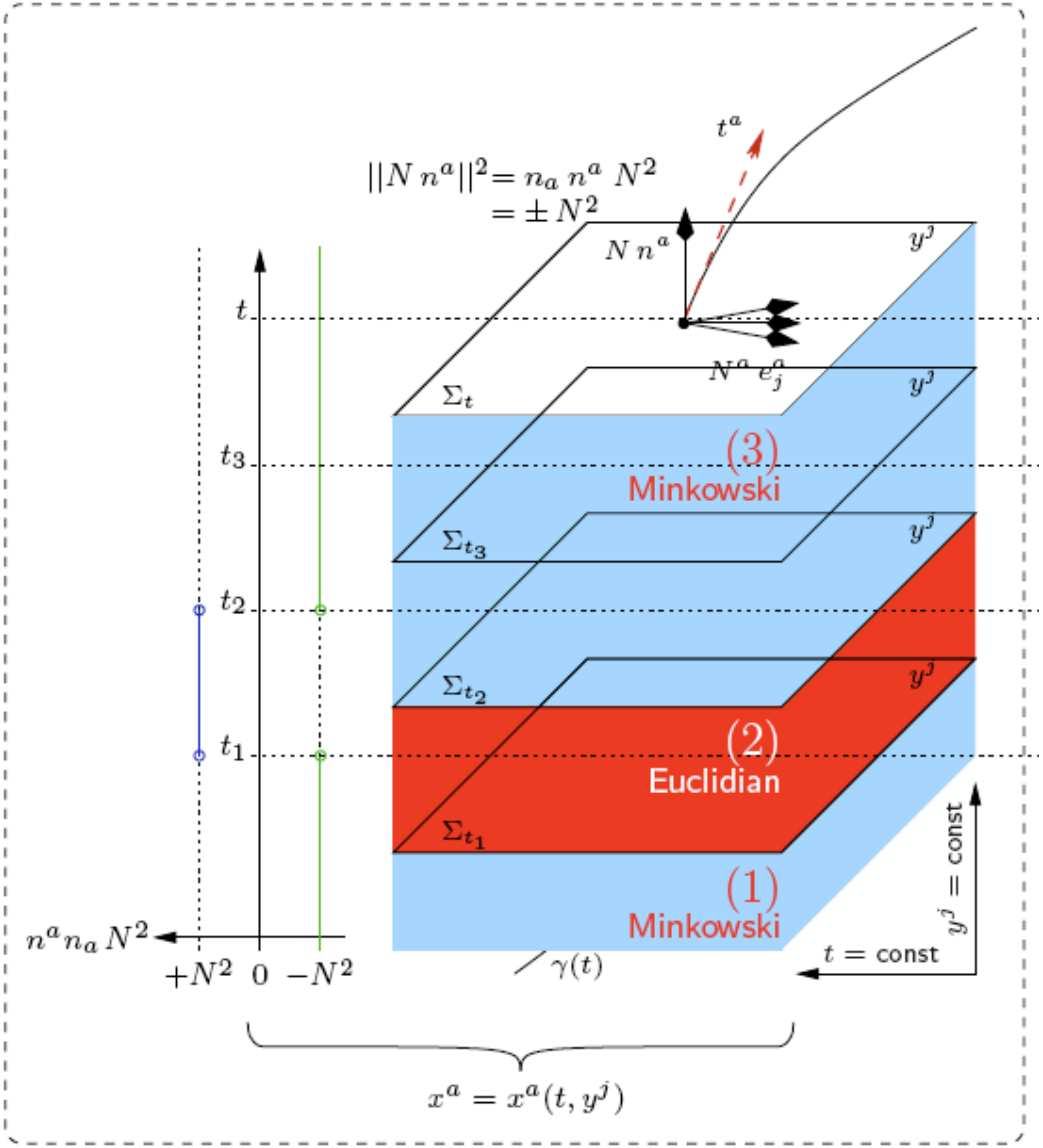

Daily signature change events...

Can we understand the bosenova experiment via the emergent spacetime programme?

Early Universe Quantum Processes in BEC Collapse Experiments

E. A. Calzetta[1] and B. L. Hu[2] *

[1]Departamento de Fisica, FCEyN Universidad de Buenos Aires Ciudad Universitaria, 1428 Buenos Aires, Argentina

[2]Department of Physics, University of Maryland, College Park, MD 20742, USA

(March 11, 2005)

- *Invited Talk presented at the Peyresq Meetings of Gravitation and Cosmology, 2003. To appear in Int. J. Theor. Phys.*

Main Theme We show that in the collapse of a Bose-Einstein condensate (BEC) [1] certain processes involved and mechanisms at work share a common origin with corresponding quantum field processes in the early universe such as particle creation, structure formation and spinodal instability. Phenomena associated with the controlled BEC collapse observed in the experiment of Donley et al [2] (they call it 'Bose-Nova', see also [3]) such as the appearance of bursts and jets can be explained as a consequence of the squeezing and amplification of quantum fluctuations above the condensate by the dynamics of the condensate. Using the

Need to understand particle production process via sudden variations in atomic-interactions...

Physical grasp on quantum field on Riemannian manifolds – super-Hubble horizon modes in cosmology:

Mechanism responsible for enormous particle production works analogous to cosmological particle production during inflation:

$$\ddot{v}_k(t) + \Omega_{\mathrm{eff}}^2\, v_k(t) = 0,$$

$$\Omega_{\mathrm{flat}}^2 = -\left(\frac{k^2}{A} + m^2\right)$$

Quantum field theory on Riemannian manifolds

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$$\ddot{\hat{\chi}}_k(t) + \left(\frac{k^2}{e^{2Ht}} + m^2 - \frac{d^2 H^2}{4} \right) \hat{\chi}_k(t) = 0$$

$$m < d\,\frac{H}{2}$$

$$k < k_{\mathrm{HubbleHorizon}}$$

$$\ddot{v}_k(t) + \Omega^2_{\mathrm{eff}}\, v_k(t) = 0,$$

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frozen modes

$$\ddot{v}_k(t) + \Omega^2_{\mathrm{eff}}\, v_k(t) = 0,$$

$$\Omega^2_{\mathrm{flat}} = -\left(\frac{k^2}{A} + m^2\right)$$

all modes are *frozen*

Significant particle production on ALL scales!???

QFT QUEEN'S FILM THEATRE

Trans-Planckian beats signature

Hydrodynamic approximation: Variations in the kinetic energy of the condensate are considered to be negligible, compared to the internal potential energy of the Bosons.

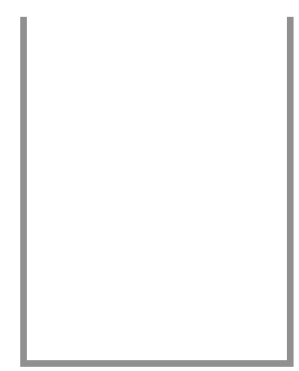
$$\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n_0+\hat{n}}}{\sqrt{n_0+\hat{n}}} \ll U$$

$$\hat{H} = \int dx \left(-\hat{\Psi}^\dagger \frac{\hbar^2}{2m}\nabla^2\hat{\Psi} + \hat{\Psi}^\dagger V_{ext}\hat{\Psi} + \frac{U}{2}\hat{\Psi}^\dagger\hat{\Psi}^\dagger\hat{\Psi}\hat{\Psi} \right)$$

Keeping quantum pressure term leads to “effective interaction” seen by inner observer:

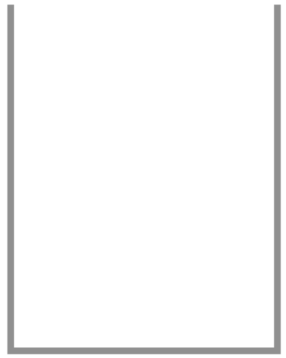
$$\mathcal{U} = U - \frac{\hbar^2}{4mn_0}\left\{ \frac{(\nabla n_0)^2 - (\nabla^2 n_0)n_0}{n_0^2} - \frac{\nabla n_0}{n_0^2}\nabla + \nabla^2 \right\}$$

harmonic trap
[position dependent sound speed]



condensate in box
[uniform number density]

Trans-Planckian beats signature



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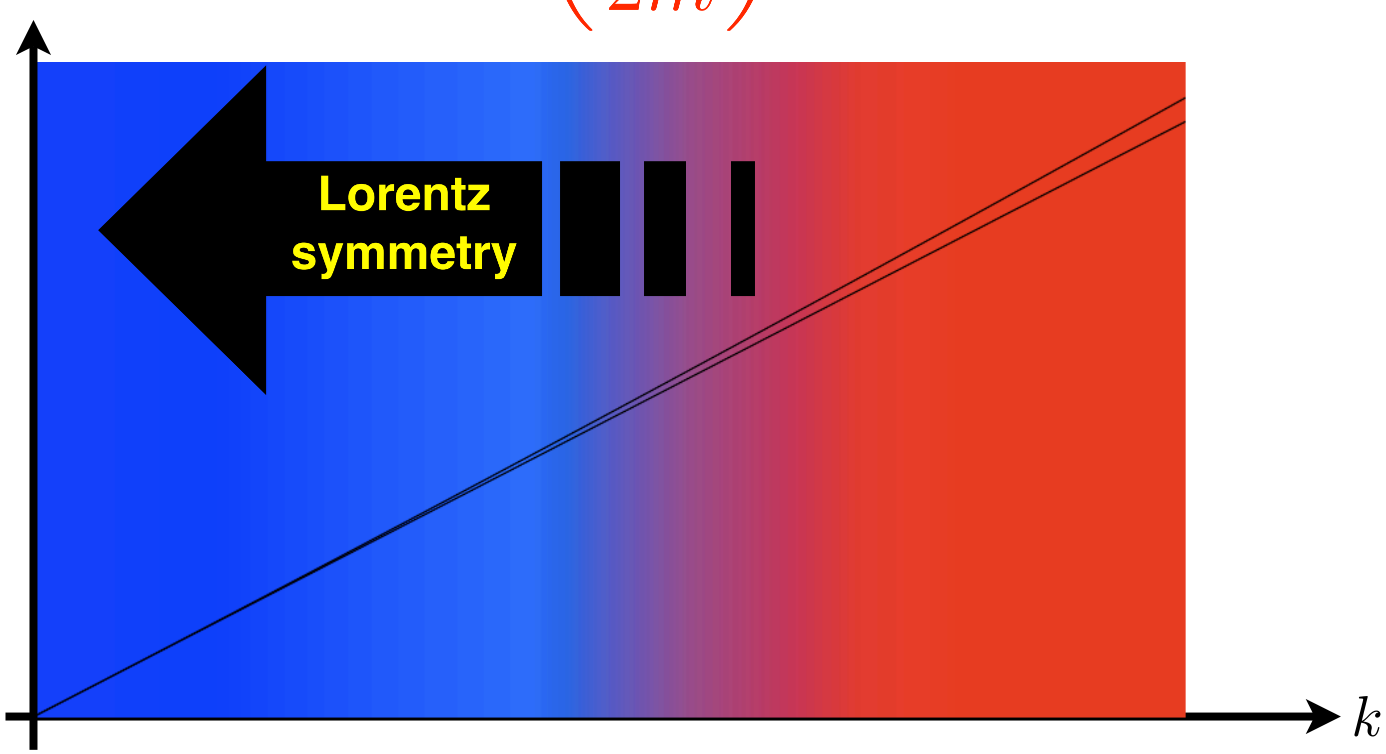
$$\mathcal{U} = U - \frac{\hbar^2}{4mn_0}\left\{\frac{(\nabla n_0)^2 - (\nabla^2 n_0)n_0}{n_0^2} - \frac{\nabla n_0}{n_0}\nabla + \nabla^2\right\} \longrightarrow \mathcal{U} = U - \frac{\hbar^2}{4mn_0}\nabla^2 \,.$$

$$c^2(U) = \frac{n_0\,U}{m} \rightarrow c_k^2(\mathcal{U}) = c^2(U) + \left(\frac{\hbar}{2m}\right)^2 k^2$$

$$\omega_k^2 = c(t)\,k^2 + \left(\frac{\hbar}{2m}\right)^2 k^4$$



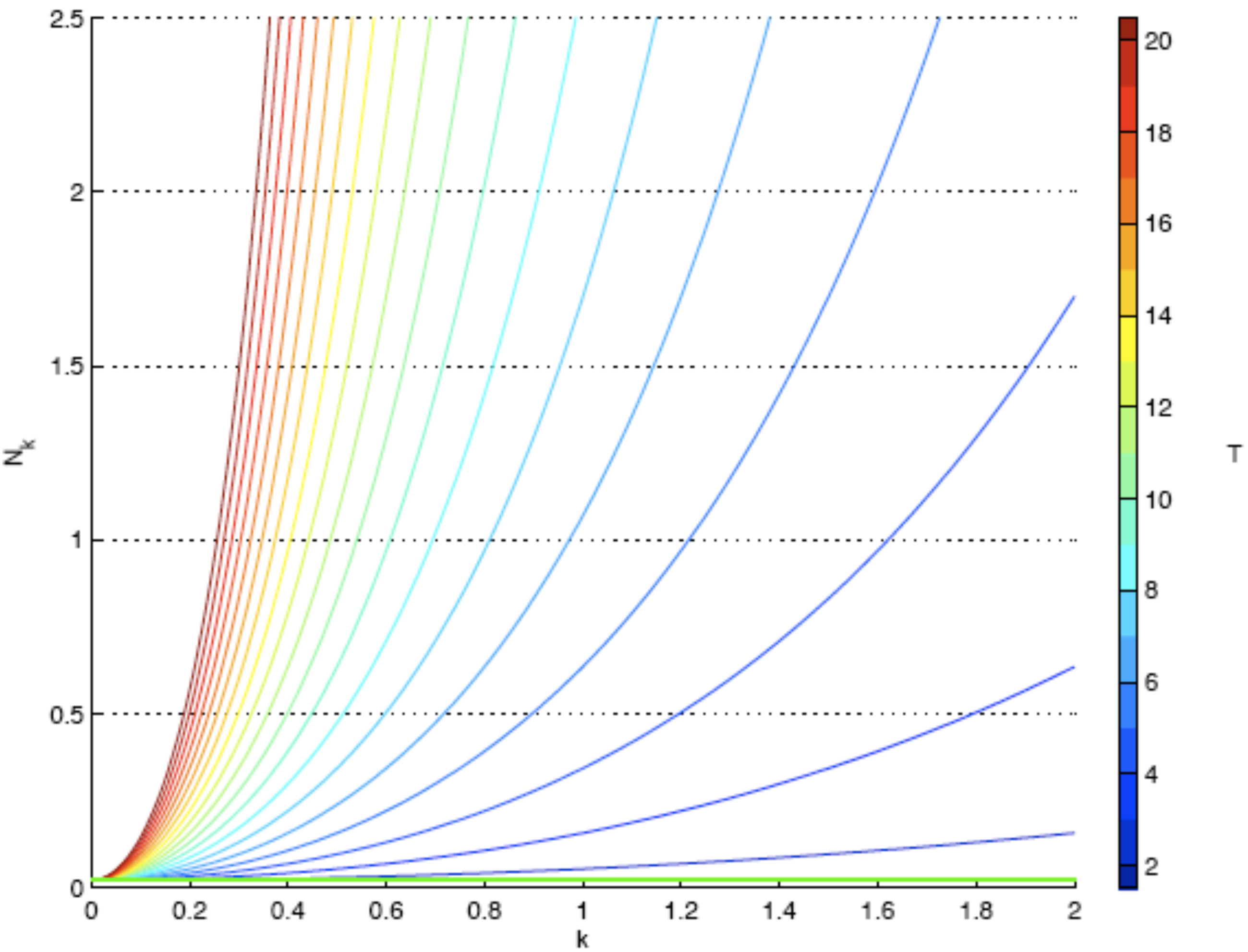
$$\omega_k \approx c(t)\,k \qquad \omega_k \approx \frac{\hbar}{2m}k^2$$

healing length:

$$\xi^2(t) = \left(\frac{\epsilon_{qp}}{c(t)}\right)^2 = \left(\frac{\hbar/2m}{c(t)}\right)^2$$

Trans-Planckian beats signature

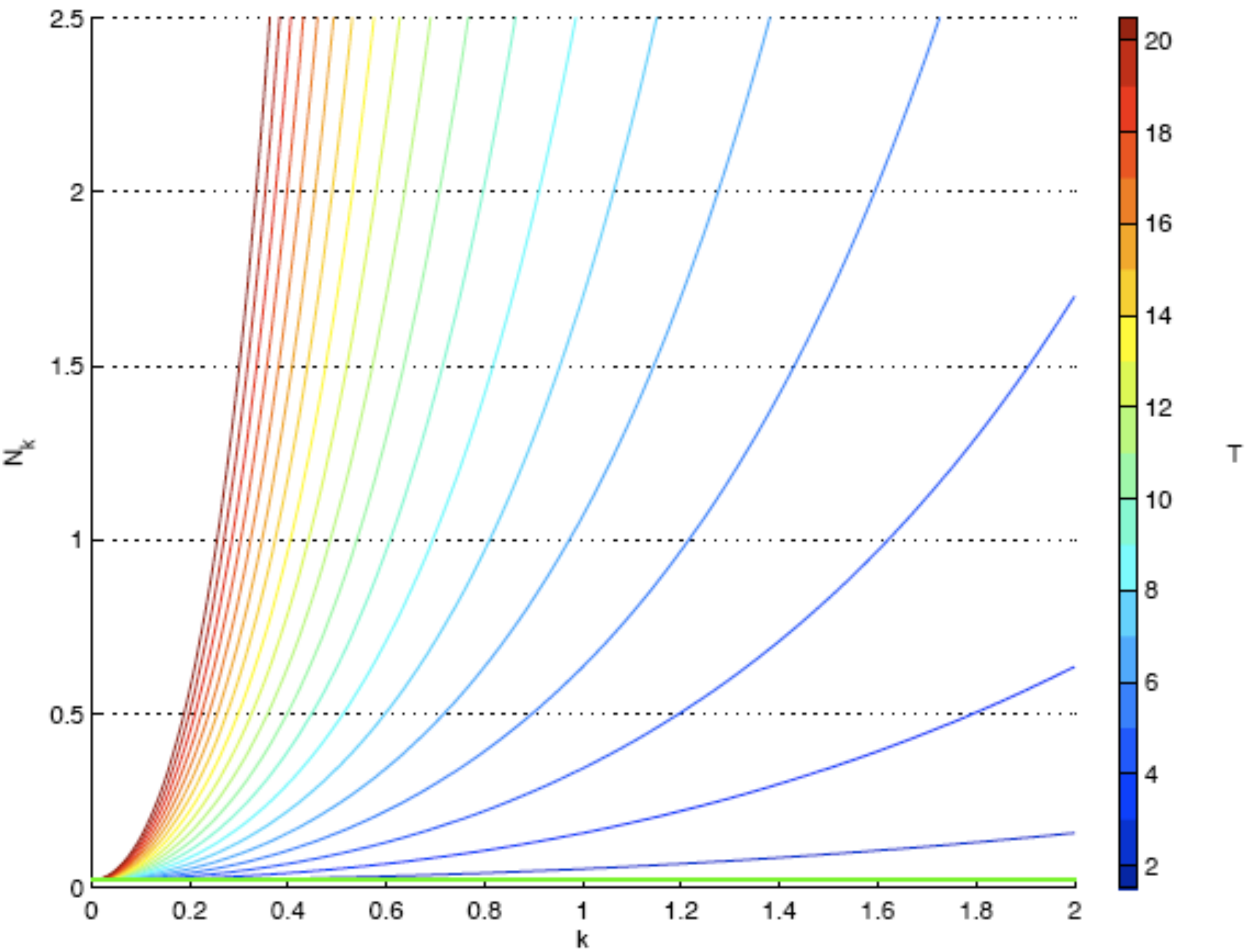
hydrodynamic approximation



Number of quasiparticles infinite!?

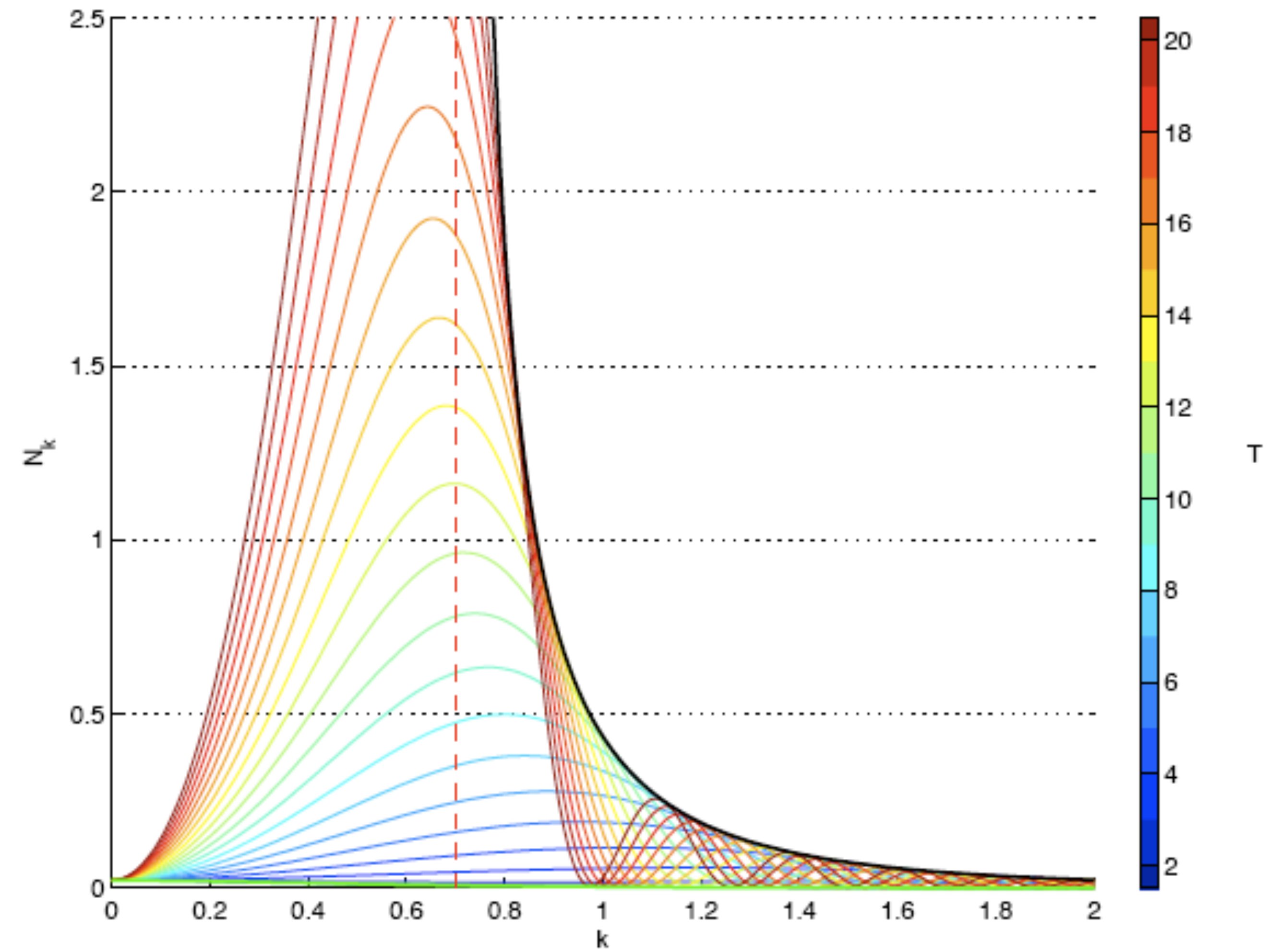
Trans-Planckian beats signature

hydrodynamic approximation



modified hydrodynamics

[including quantum pressure effects]



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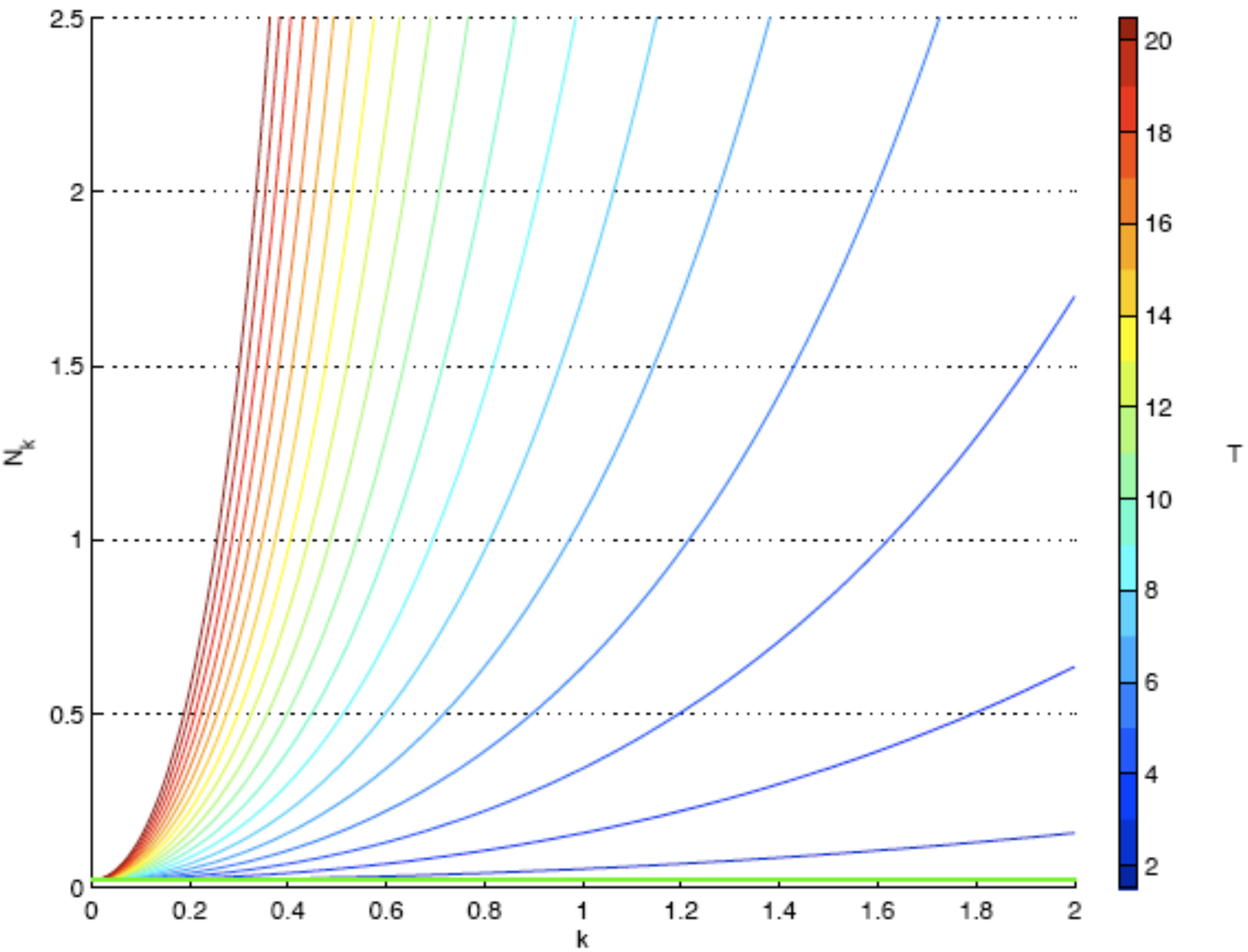

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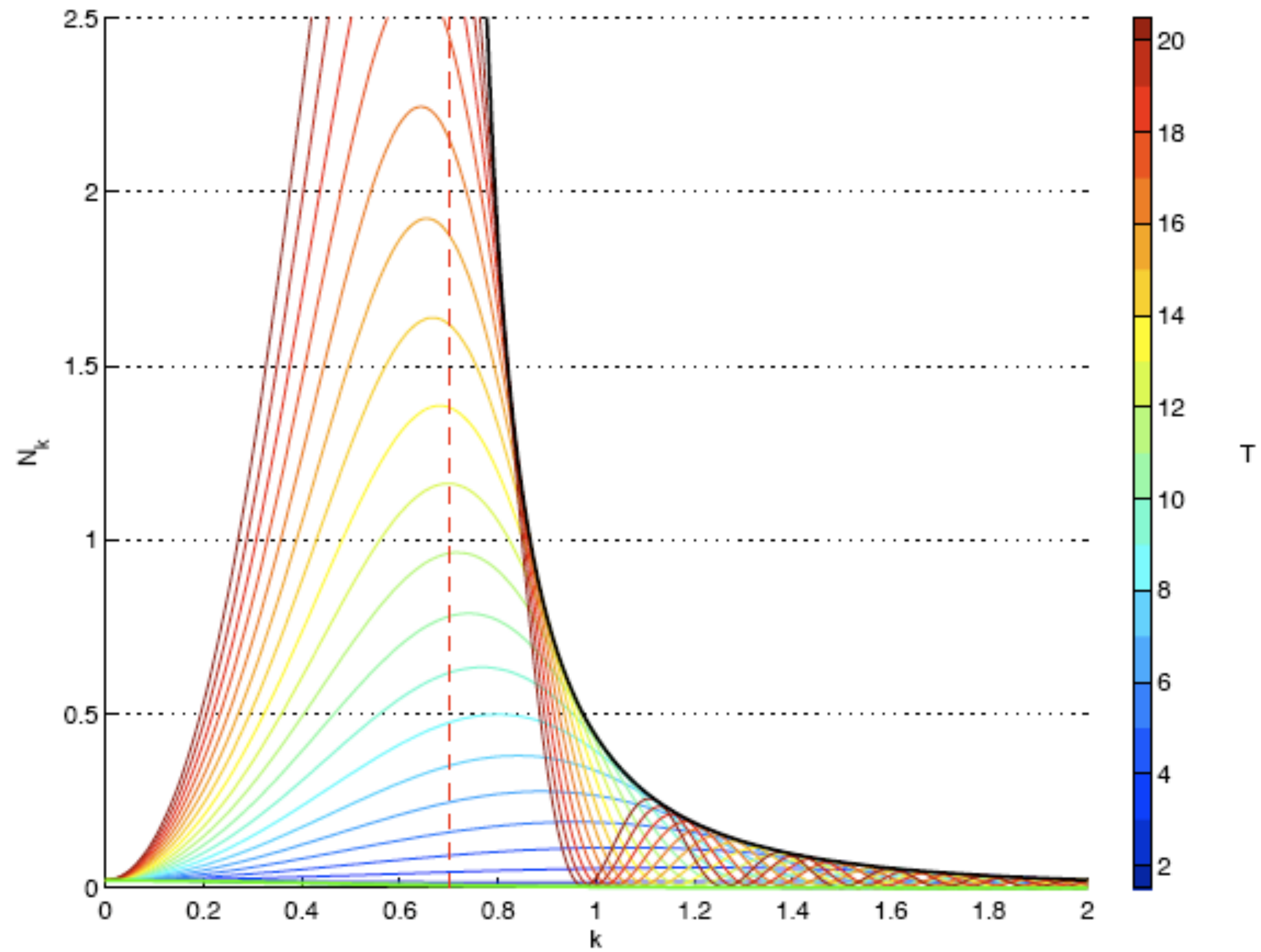
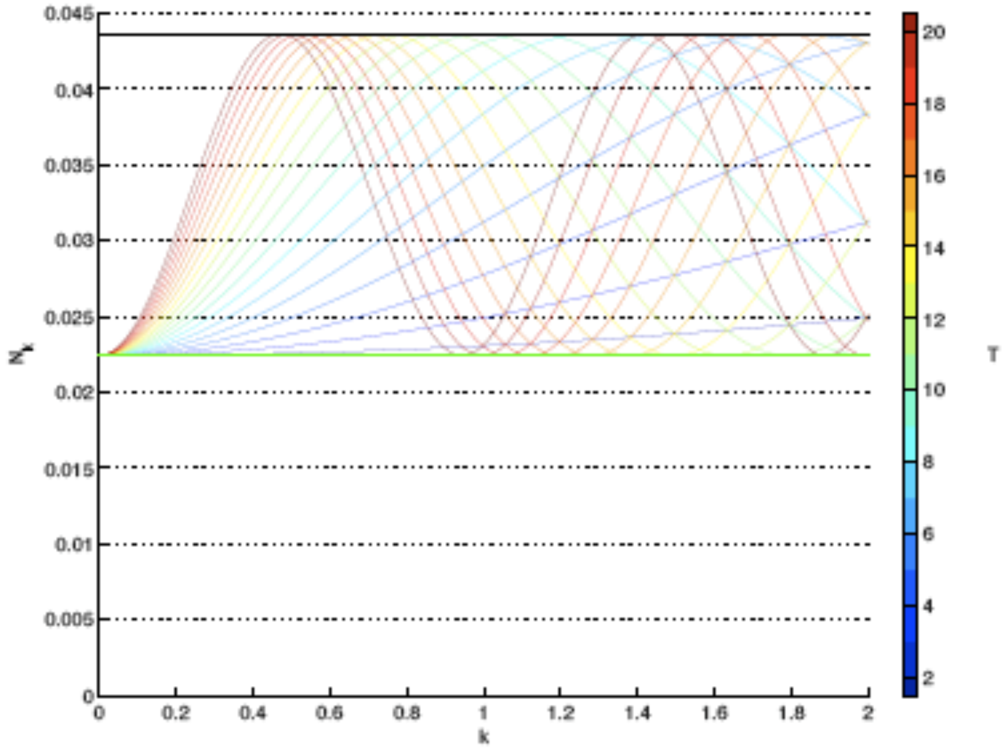
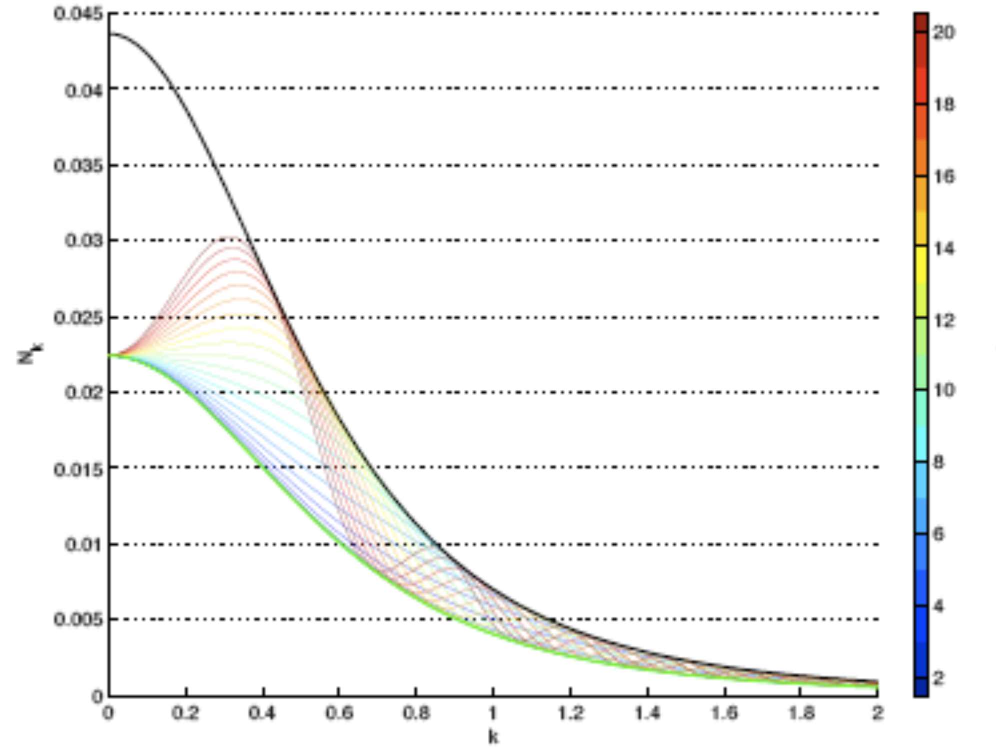


$$\mathcal{U}|_{\nabla \to -ik} \to U_k = U + \frac{\hbar^2}{4mn_0}k^2$$

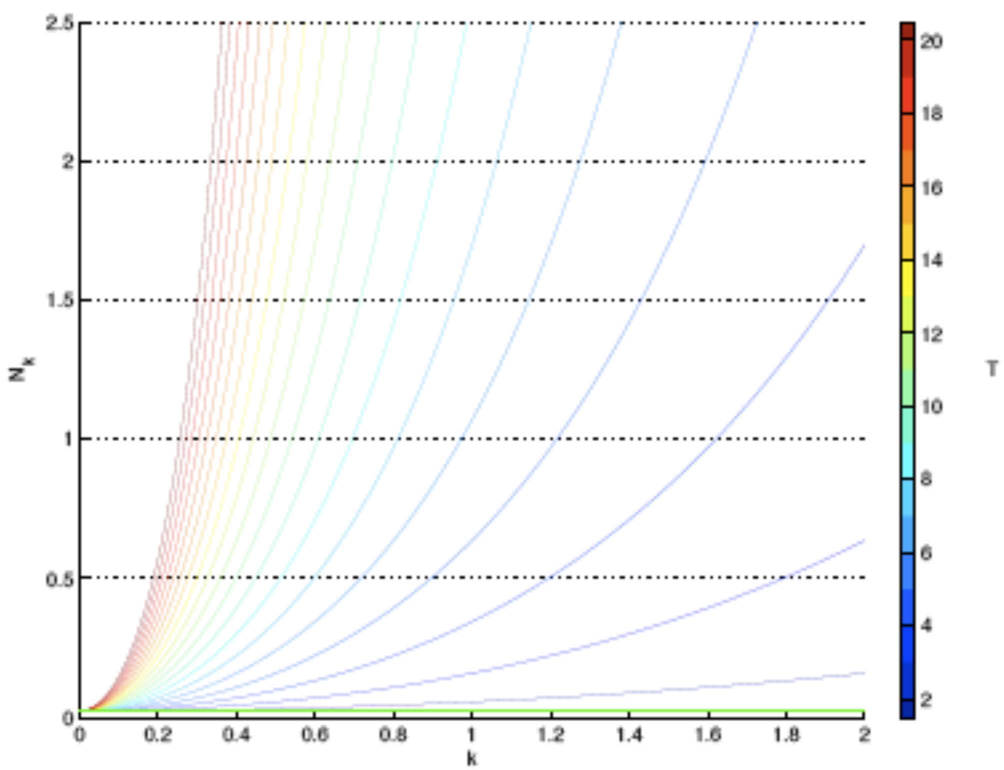
Trans-Planckian beats signature



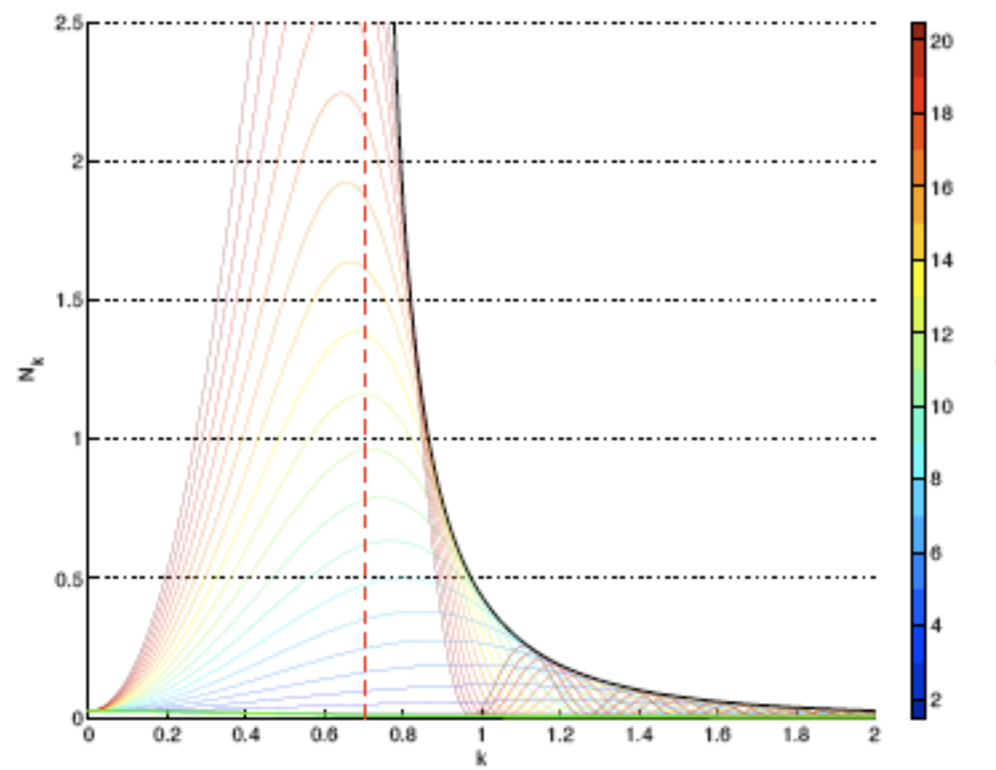
(a) Hydrodynamic limit; L-L-L.



(b) Microscopic corrections; L-L-L.

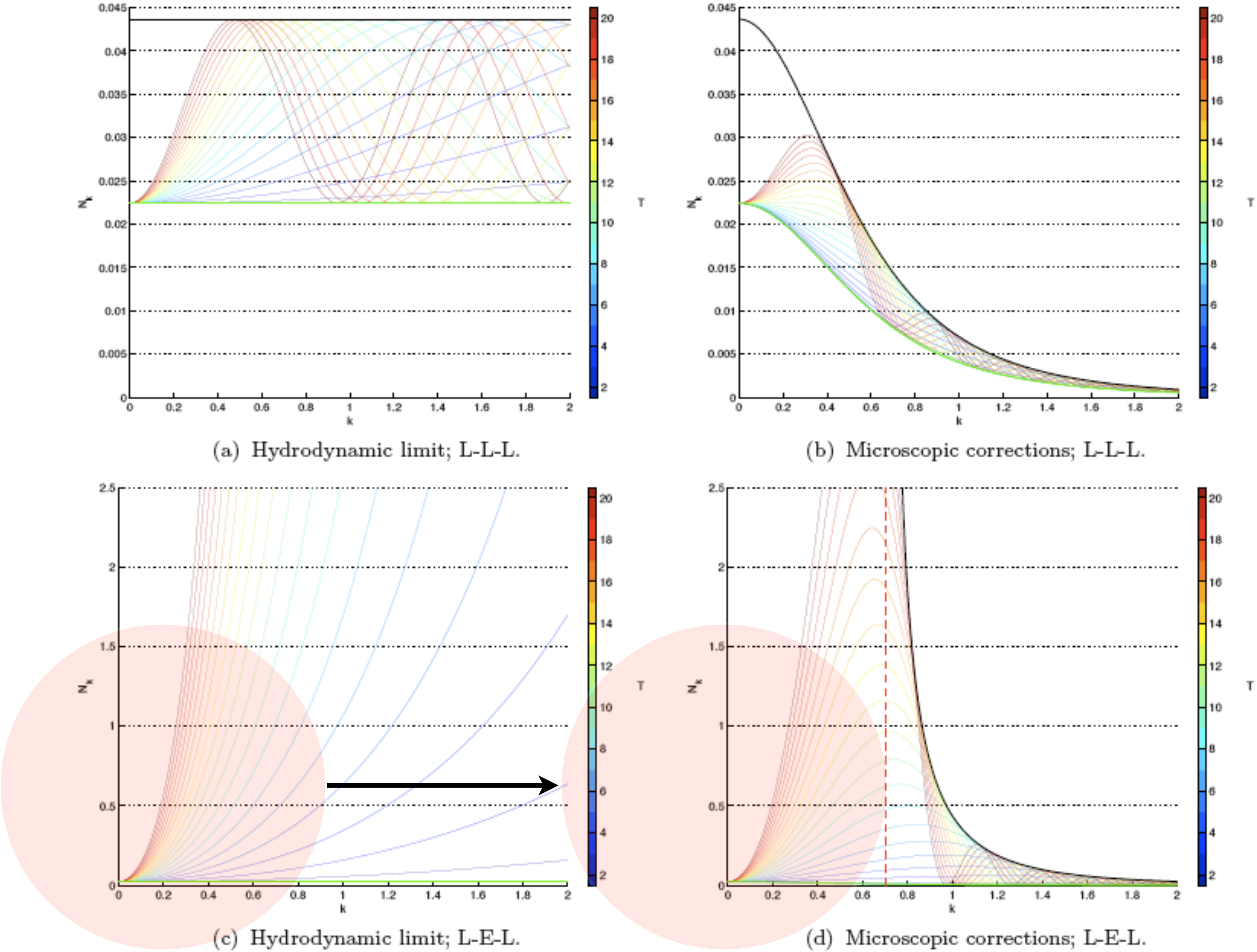


(c) Hydrodynamic limit; L-E-L.



(d) Microscopic corrections; L-E-L.

Trans-Planckian beats signature



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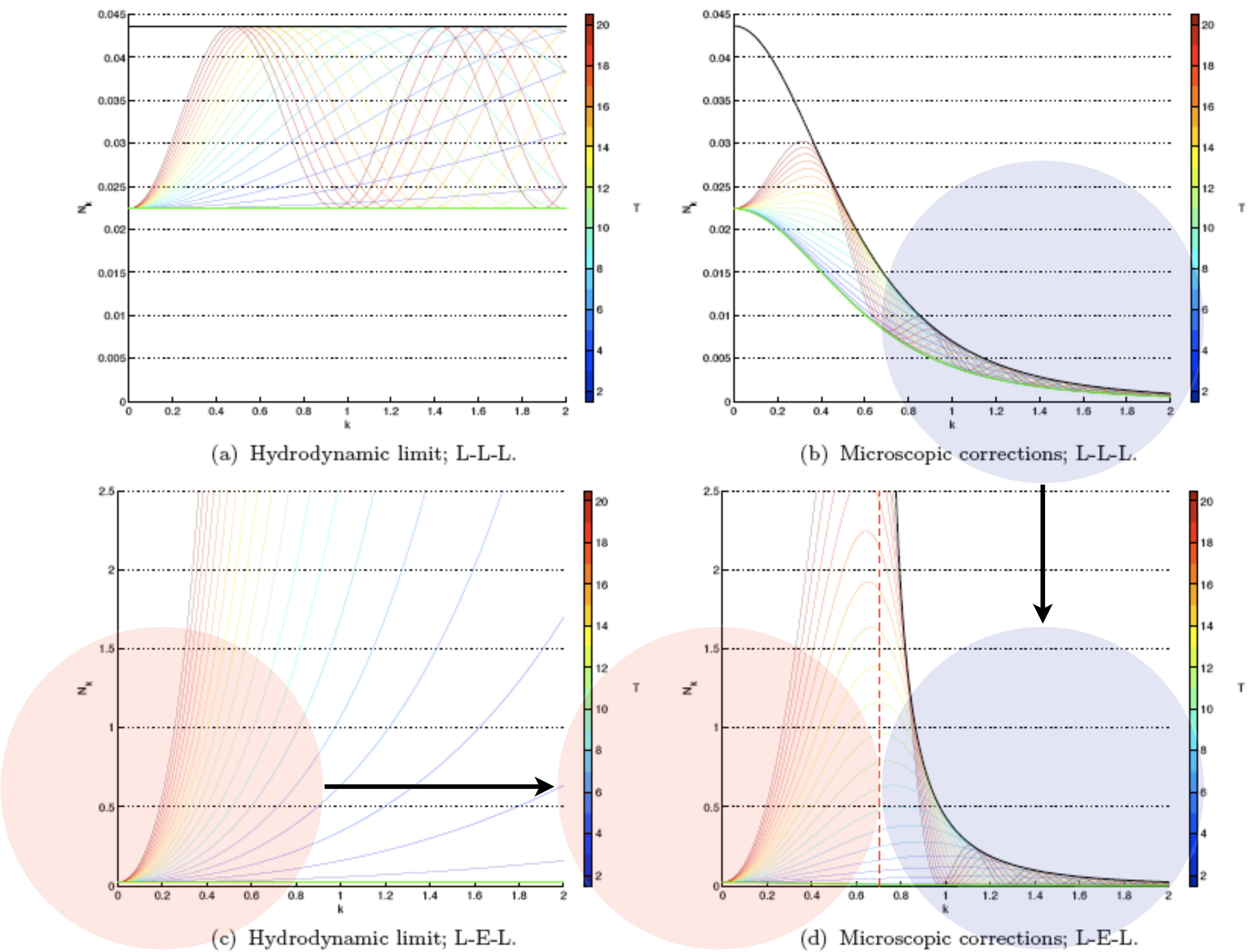

Trans-Planckian beats signature - but why?

Let's do quantum gravity phenomenology, in the sense of an ultra-high energy breakdown of Lorentz symmetry

T. Jacobson, "Black hole evaporation and ultrashort distances", Phys. Rev. D 44 (1991) 1731

$$\Delta_{d+1}\phi - F(-\Delta_d)\phi = m^2\phi$$

where

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$$\bar{\omega}^2_{\text{effective}} = \epsilon A^d\left[m^2 + F(k^2/A) + k^2/A\right]$$

$$\beta \approx i\sinh\left\{\int_E \sqrt{m^2 + k^2/A + F(k^2/A)}\;A^{d/2}\,d\bar{t}\right\}$$

Trans-Planckian beats signature

Particle production in *real* world with naive LIV terms:

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Particle production in analogue *world* - a BEC - with quantum pressure correction to the mean-field:

$$\beta \approx i \sin \left\{ \int_E \sqrt{B\, m^2 + \varepsilon_{\rm qp}^2\, k^4 + B\, k^2/A} \; A^{d/2} \; d\bar{t} \right\}$$

Conclusions for signature change events

* quantum modes on a Riemannian manifold have like super-Hubble horizon modes during inflation
=> Explains particle production

* Signature change events in the *real* universe show serious problems: driving the production of an infinite number of particle, with infinite energy, which are not removed by dimension, rest mass, or even reasonable sub-class of LIV

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If there is a way to drive sig. change events within the realm QG, there should be a mechanism to regularize the infinities..! (Analogue to the situation in the BEC)

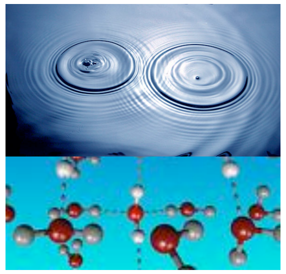
Quantum gravity phenomenology

Quantum gravity phenomenology [LIV]

QGP: Summarizes all possible **<u>phenomenological consequences from quantum gravity</u>**. While different quantum gravity candidates may have completely distinct physical motivation, they can yield similar observable consequences, e.g. **<u>Lorentz symmetry breaking at high energies</u>**.

1) Presense of a preferred frame;
2) All frames equal, but transformation laws between frames are modified.

D. Mattingly. Modern tests of Lorentz invariance. *Living Reviews in Relativity*, 8(5), 2005.



Analogue Lorentz symmetry breaking

Symmetry breaking mechanism in different analogue models lead to model-specific modifications:

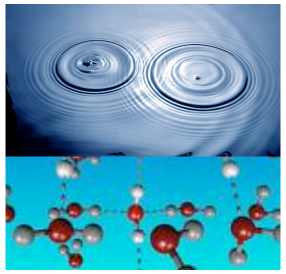
Bose-Einstein condensate: $\omega_k^2 = c_0^2\, k^2 + \epsilon_{\rm qp}^2\, k^4$

Electromagnetic waveguide: $\omega_k^2 = \frac{4}{L\,C}\,\sin^2\left(\frac{k\,\Delta x}{2}\right) \approx c^2\,k^2 - \frac{\Delta x^4}{12\,L\,C}\,k^4\,.$

Despite all fundamental differences similar modifications:

$$\Delta\omega_k^2 \sim \pm k^4.$$

Any emergent spacetimes based on analogue models per definition have a preferred frame: The external observer.



Generating a *mass* and Goldstone's theorm

[*] Back to the equation of motion -

Can we extend the class of fields...

[*] Goldstone's theorem -

Spontaneous symmetry breaking...

[*] Mass generating mechnism -

Explicit symmetry breaking...

Goldstone's theorem...

"... whenever a continuos symmetry is spontaneously broken, massless fields, known as Nambu-Goldstone bosons, emerge." [Quantum Field Theory in a Nutshell, A. Zee]

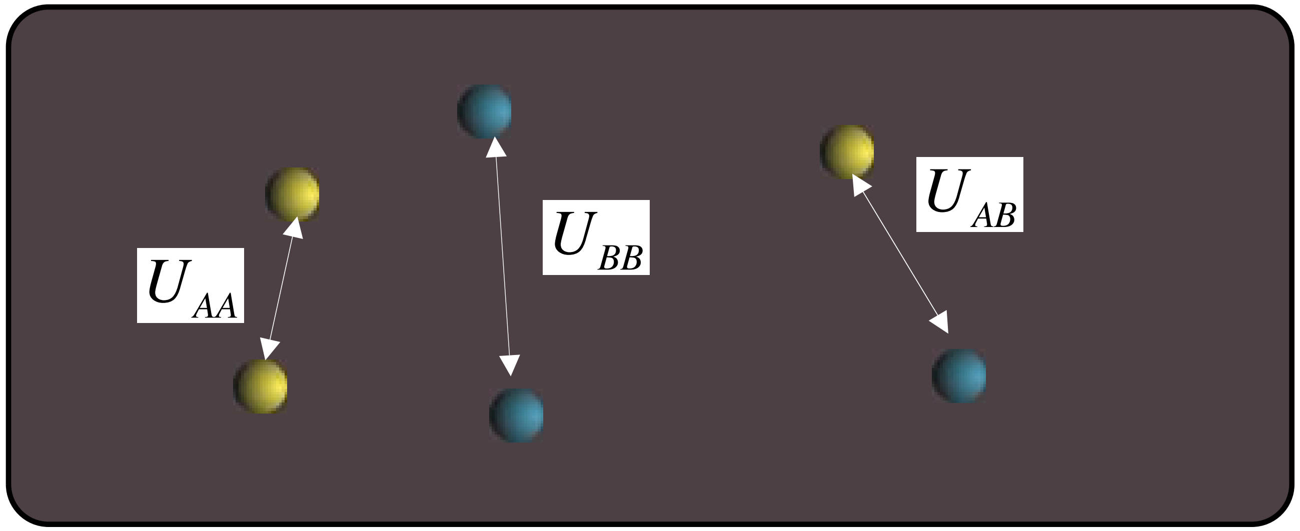
Per definition Bose-Einstein condensation always (spontaneously) breaks SO(2) symmetry of many-body Hamiltonian!!!

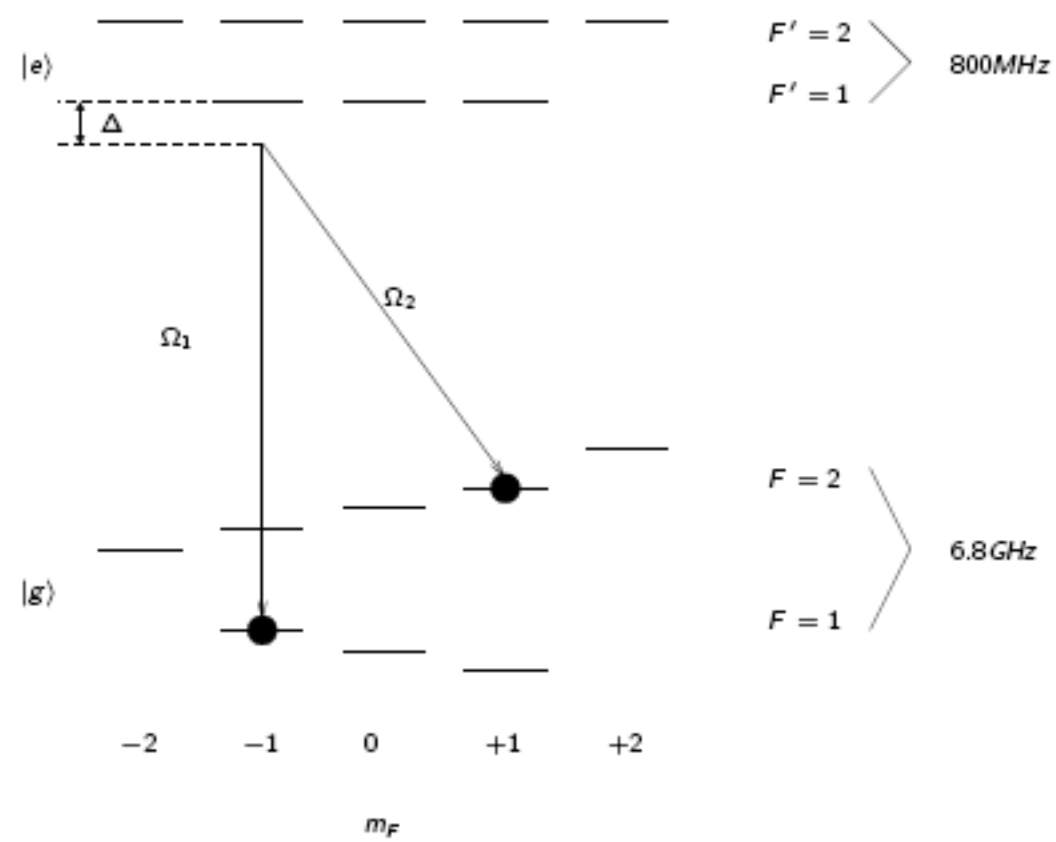
$$\hat{H} = \int d\mathbf{x} \left(-\hat{\Psi}^\dagger \frac{\hbar^2}{2m} \nabla^2 \hat{\Psi} + \hat{\Psi}^\dagger V_{\mathbf{ext}} \hat{\Psi} + \frac{U}{2} \hat{\Psi}^\dagger \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi} \right)$$

$\mathrm{SO}(2) - \mathrm{symmetry}$ $\quad \hat{\Psi} \rightarrow \hat{\Psi}^* = \hat{\Psi} \exp(i\alpha)$

Mass generating mechnism - all about symmetries

M. Visser and S. W. *Phys. Rev.*, D72:044020, 2005.





Explicit symmetry breaking through transitions in a 2-component system

$$\hat{H} = \int \mathrm{d}\mathbf{r} \left\{ \sum_{i=1,2} \left(-\hat{\Psi}_i^\dagger \frac{\hbar^2 \nabla^2}{2m_i} \hat{\Psi}_i + \hat{\Psi}_i^\dagger V_{ext,i}(\mathbf{r}) \hat{\Psi}_i \right) + \frac{1}{2} \sum_{i,j=1,2} \left(U_{ij} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_i \hat{\Psi}_j + \lambda \hat{\Psi}_i^\dagger (\sigma_x)_{ij} \hat{\Psi}_j \right) \right\}$$

$$SO(2)_A \times SO(2)_B \to S0(2)_{AB}$$

$$\theta_A \to \tilde{\theta}_A = \theta_A \exp(+i\,\alpha) \qquad \theta_B \to \tilde{\theta}_B = \theta_B \exp(-i\,\alpha)$$

However, the fundamental Hamiltonian of the two-component system is a functional of $\vec{\theta} = (\theta_A, \theta_B)$. In the absence of transitions between the two fields the Hamiltonian exhibits an extra $SO(2)$ symmetry under which $\vec{\theta}$ transforms as a 2-component vector. This symmetry is explicitly broken for interacting fields, so that $SO(2)_A \times SO(2)_B \to S0(2)_{AB}$. The coupled system is only invariant under simultaneous transformations of the form $\theta_A \to \tilde{\theta}_A = \theta_A \exp(+i\,\alpha)$, and $\theta_B \to \tilde{\theta}_B = \theta_B \exp(-i\,\alpha)$. Thus the spontaneous symmetry breaking during the Bose–Einstein condensation relates to $S0(2)_{AB}$, instead of the individual symmetries. Altogether, linearizing around both fields yields two excitations, where one *has* to be a “Nambu–Goldstone Boson” (*i.e.*, a zero-mass excitation), while there are no constraints on the mass of the second quasi-particle.

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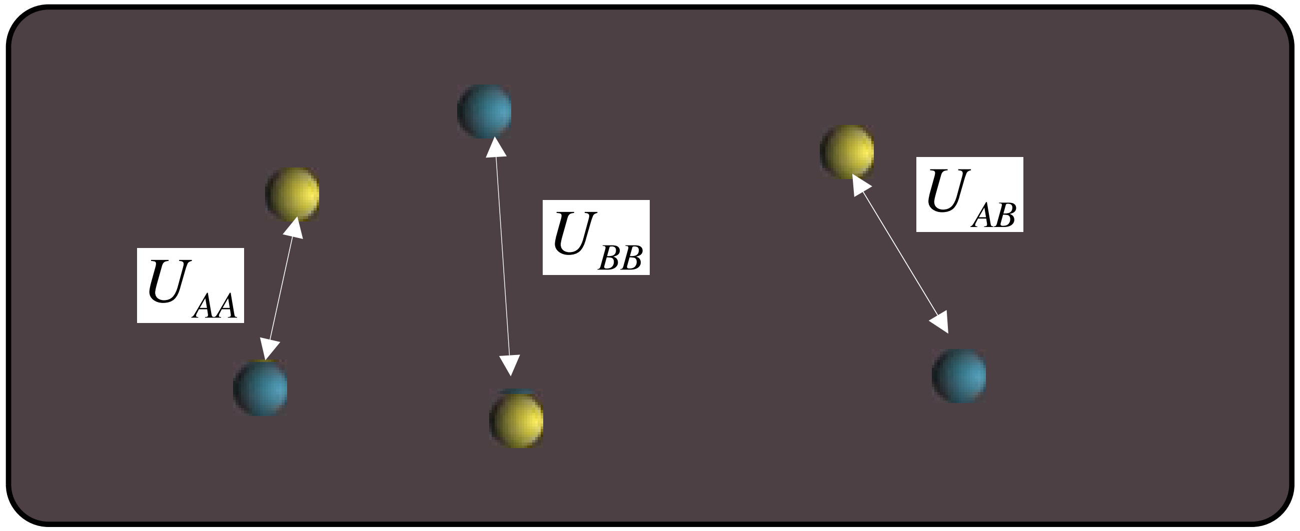

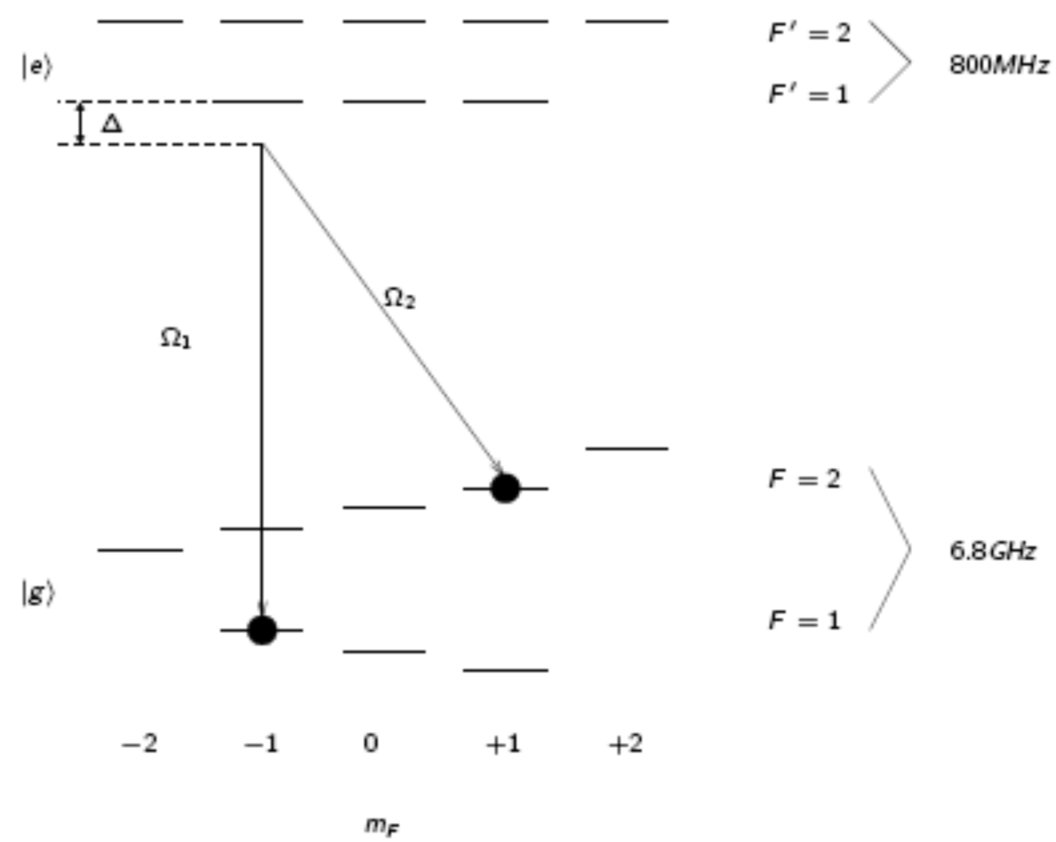

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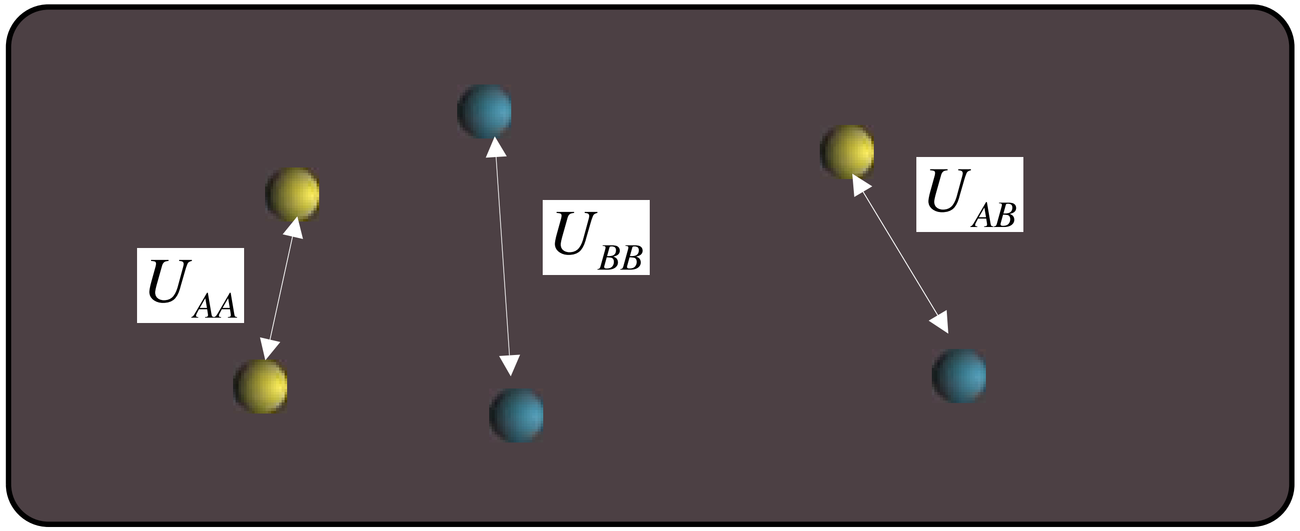

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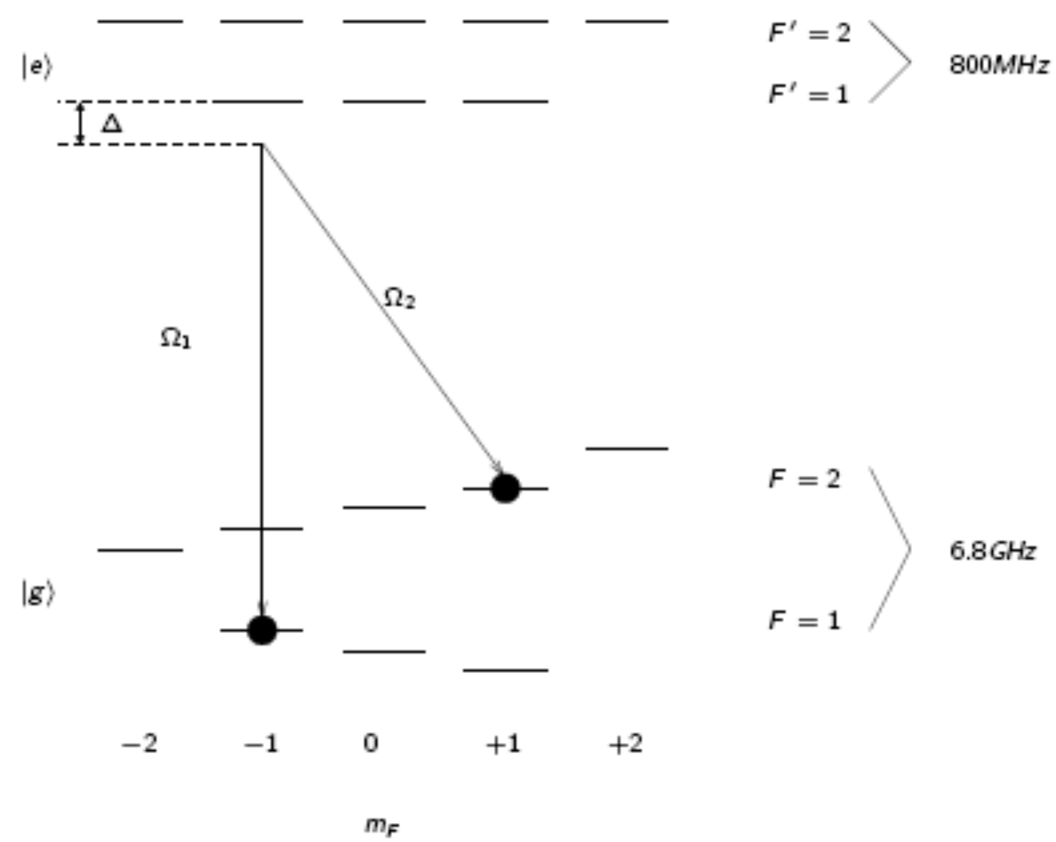

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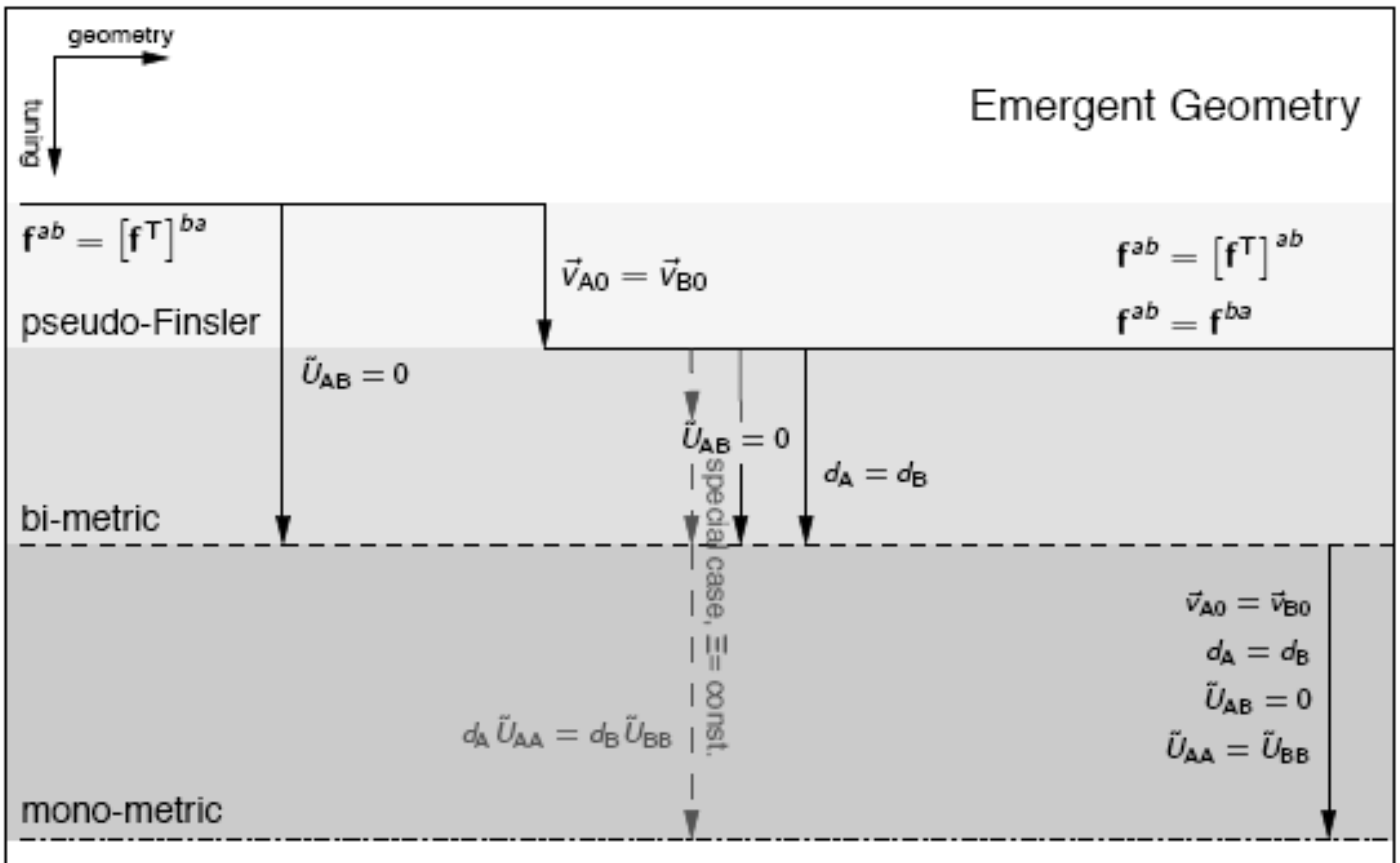
Mono-metricity

More complicated hyperbolic wave equation: $\partial_a \left(\mathsf{f}^{ab}\, \partial_b \bar\theta\right) + (\Lambda + K)\ \bar\theta + \frac{1}{2}\left\{\Gamma^a\, \partial_a \bar\theta + \partial_a(\Gamma^a \bar\theta)\right\} = 0$

$$\mathsf{f}^{ab} = \left[\begin{array}{c|c} \Xi_{11}^{-1}\left(\begin{array}{c|c} -1 & -\vec{V}_{A0}^{T} \\ \hline -\vec{V}_{A0} & \frac{d_A}{\Xi_{11}^{-1}}\delta_{ij} - \vec{V}_{A0}\vec{V}_{A0}^{T} \end{array}\right) & \Xi_{12}^{-1}\left(\begin{array}{c|c} 1 & \vec{V}_{B0}^{T} \\ \hline \vec{V}_{A0} & \vec{V}_{A0}\vec{V}_{B0}^{T} \end{array}\right) \\ \hline \Xi_{21}^{-1}\left(\begin{array}{c|c} 1 & \vec{V}_{A0}^{T} \\ \hline \vec{V}_{B0} & \vec{V}_{B0}\vec{V}_{A0}^{T} \end{array}\right) & \Xi_{22}^{-1}\left(\begin{array}{c|c} -1 & -\vec{V}_{B0}^{T} \\ \hline -\vec{V}_{B0} & \frac{d_B}{\Xi_{22}^{-1}}\delta_{ij} - \vec{V}_{B0}\vec{V}_{B0}^{T} \end{array}\right) \end{array}\right]$$

$$\mathsf{f}^{ab} \sim \sqrt{-g}\, g^{ab}$$



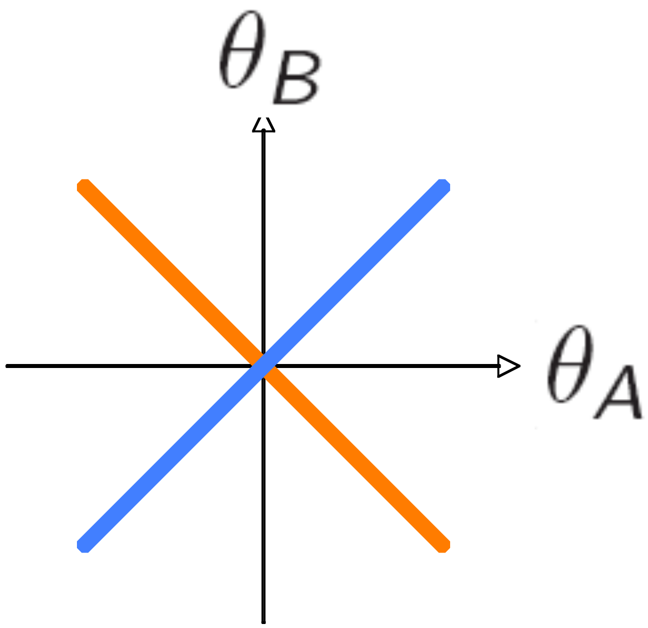
$$ds = \sqrt[4]{g_{abcd}\, dx^a dx^b dx^c dx^d}$$

Emergent massive fields

$$\frac{1}{\sqrt{-g_{I/II}}}\partial_a\left\{\sqrt{-g_{I/II}}\,(g_{I/II})^{ab}\,\partial_b\tilde{\theta}_{I/II}\right\}+\omega^2_{I/II}\,\tilde{\theta}_{I/II}=0$$

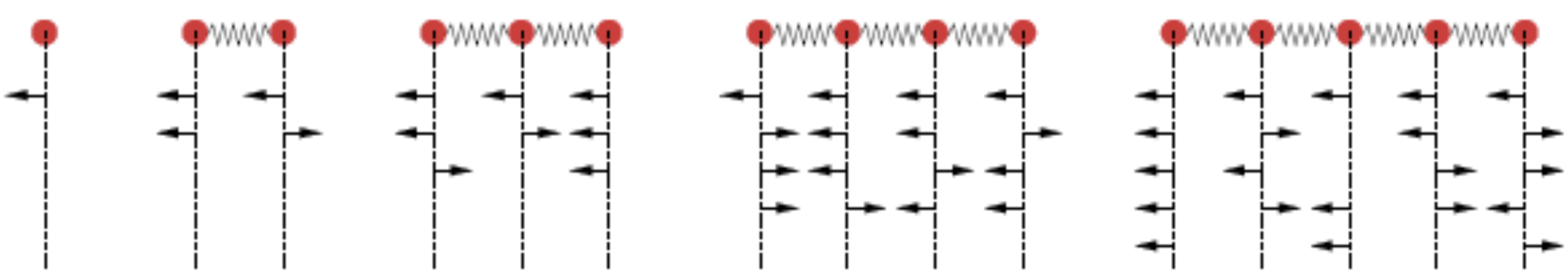
the acoustic metrics are given by

$$(g_{I/II})_{ab}\propto\left[\begin{array}{c|c}-(c^2-v_0^2) & -\vec{v_0}^{\,T}\\ \hline -\vec{v_0} & \mathbf{I}_{d\times d}\end{array}\right]$$



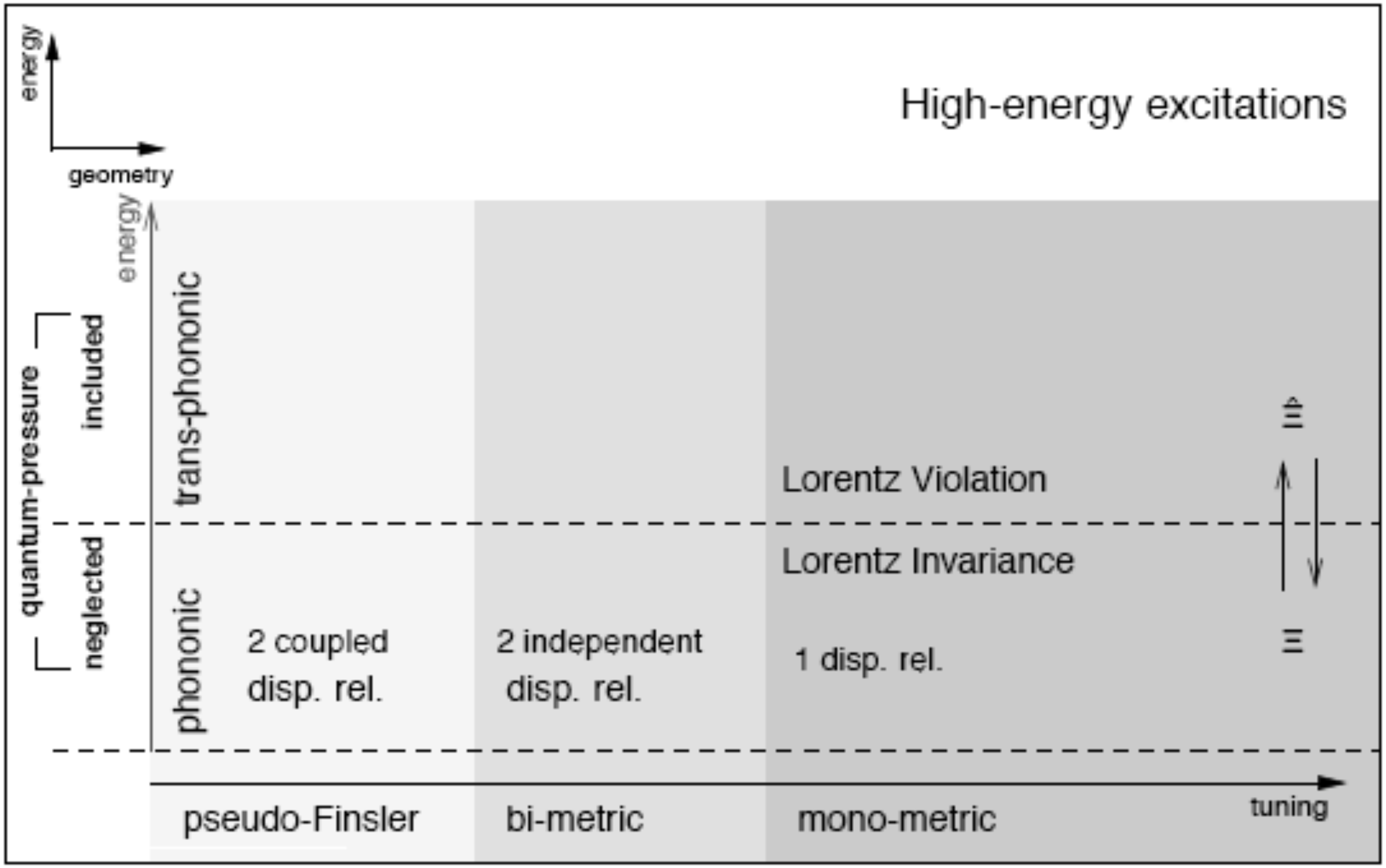
In-phase perturbtion (= mass zero particle): $\omega_I^2=0$

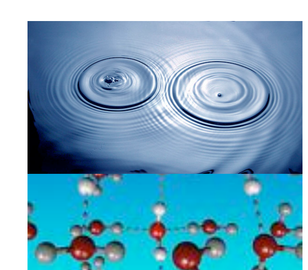
Anti-phase perturbtion (= mass non-zero particle): $\omega_{II}^2\propto\lambda\neq 0$



Collective excitations + microscopic fingerprints

Beyond the hydrodynamic approximation...





[1] S. Liberati, M. Visser, and S. W.. *Class. Quant. Grav.*, 23:3129–3154, 2006.

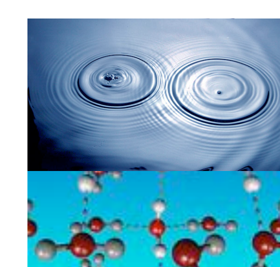
[2] S. Liberati, M. Visser, and S. W.. *Phys. Rev. Lett.*, 96:151301, 2006.

Bogoliubov dispersion realtion for 2-comp. sys.

$$\omega_k^2 = \omega_0^2 + (1+\eta_2)\, c^2 k^2 + \eta_4 \left(\frac{\hbar}{M_{\mathrm{LIV}}}\right)^2 k^4 + \cdots$$

$$\eta_{4,X} \approx 1; \qquad X = \mathrm{I}, \mathrm{II}$$

$$\eta_{2,X} \approx \left(\frac{m_X}{M_{\mathrm{eff}}}\right)^2 = \left(\frac{\text{mass scale of quasiparticle}}{\text{effective Planck scale}}\right)^2; \qquad X = \mathrm{I}, \mathrm{II}$$



Naturalness problem in emergent spacetime

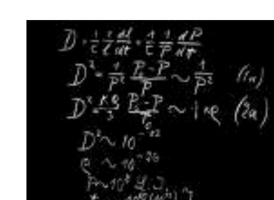
Dispersion relation obtained from our CMS has the form:

$$\omega^2 = \omega_0^2 + (1+\eta_2)\ c^2\ k^2 + \eta_4 \left(\frac{\hbar}{M_{\mathrm{LIV}}}\right)^2 k^4 + \ldots$$

* CPT invariant (LIV in the boost subgroup)
* has the form as suggested in many (non-renormalizable) effective field theory approaches
* **natural suppression of low-order modifications in our model!**

$$\eta_{2,\mathrm{I/II}} \approx \left(\frac{m_{\mathrm{I/II}}}{M_{\mathrm{LIV}}}\right)^2 = \left(\frac{\text{quasiparticle mass}}{\text{effective Planck scale}}\right)^2 ;$$
$$\eta_{4,\mathrm{I/II}} \approx 1;$$

* analogue LIV scale is given by the microscopic variables: $M_{\mathrm{LIV}} = \sqrt{m_A m_B}$
* not a tree-level result. results directly computed from fundamental Hamiltonian
* η_4-coefficients are different for $m_A \neq m_B$



Cosmolgoy

On robustness...

Robustness (against microscopic modifications) of quantum field theory in emergent spacetimes?

P. Jain, S. W., M. Visser, and C. Gardiner. Analogue model of an expanding FRW universe in Bose–Einstein condensates: Application of the classical field method. arXiv:0705.2077, 2006. Accepted for publication in Phys. Rev. A.

Emergent FRW universe emerging from Bose gas:

- Microscopic substructure induces Lorentz symmetry breaking as suggested by many Effective field theories, *i.e.*, preferred frame induces non-linear dispersion in the boost-subgroup.

  S. Liberati, M. Visser, and S. W.. Naturalness in emergent spacetime. *Phys. Rev. Lett.*, 96:151301, 2006.

- Particle production in general is not robust and significant modification may appear.

Application for "real" cosmology?

Are there any general lessons to be learnt from emergent FRW spacetimes - incorporating model-specific modifications - for cosmology?

"Planck"-modifications

Rainbow geometry	k-dependent commutator relation for matter fields

Modifications in:

Group and phase velocity	Dispersion relation	Cosmological horizons

Rainbow geometries

Emergent spacetime exhibits explicit momentum-dependence:

$$g_{ab} \equiv \left(\frac{n_0 \hbar}{c_k m}\right)^{\frac{2}{d-1}} \begin{bmatrix} -(c_k^2 - v^2) & -v^j \\ -v^i & \delta^{ij} \end{bmatrix}$$

Macroscopic variables determine metric components:

Speed of sound for perturbations in the condensate:

$$c_k(t)^2 = c(t)^2 + \gamma_{qp}^2 k^2$$

Where $\gamma_{qp} = \frac{\hbar}{2m}$ represents the UV correction!

Background velocity as gradient of the phase of the condensate:

$$\mathbf{v} = \frac{\hbar}{m} \nabla \theta_0 = \text{here} = \mathbf{0}$$

$$c_k^2 = c^2 \left(1 + \frac{\gamma_{qp}^2}{c^2} k^2\right) := c^2 \left(1 + k^2/K^2\right)$$

Rainbow FRW geometries

In (2+1) dimensions - for time-dependent atomic interactions - we get:

$$ds^2_{d=2} = \left(\frac{n_0}{c_0}\right)^2 \left[-c_0^2 dt^2 + b_k(t)^{-1} d\mathbf{x}^2\right]$$

Where the time- and momentum dependence enter as follows:

$$a_k(t) = b_k(t)^{-1/2} = \frac{1}{\sqrt{b(t) + (k/K)^2}}$$

The closest to a de Sitter like expansion (exact de Sitter in on infrared scales) is given by:

$$a_k(t) = \frac{1}{\sqrt{\exp(-2Ht) + (k/K)^2}}$$

Rainbow FRW geometries

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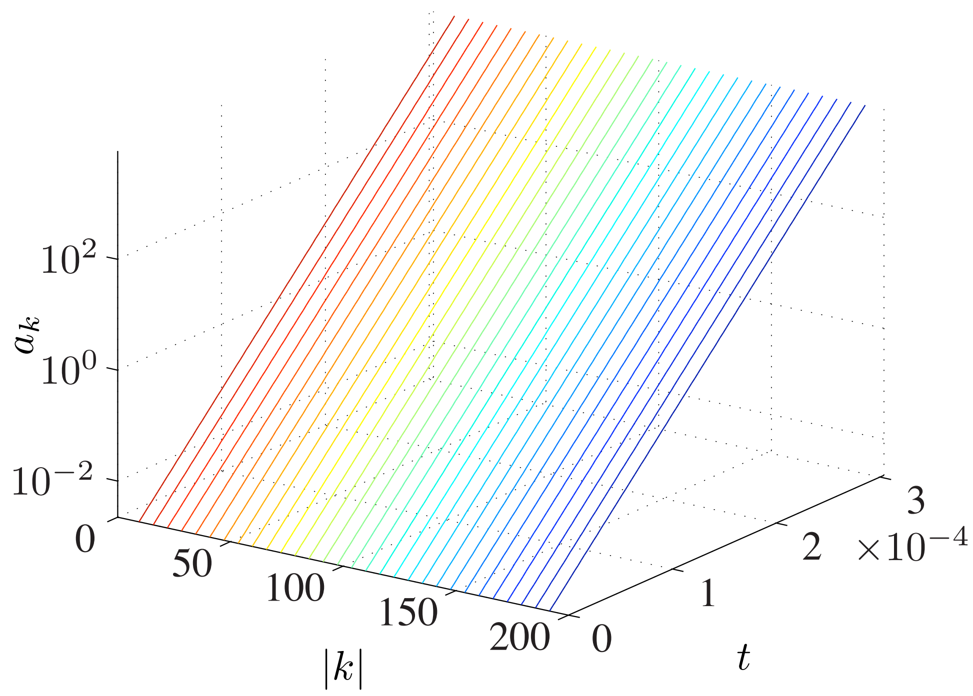
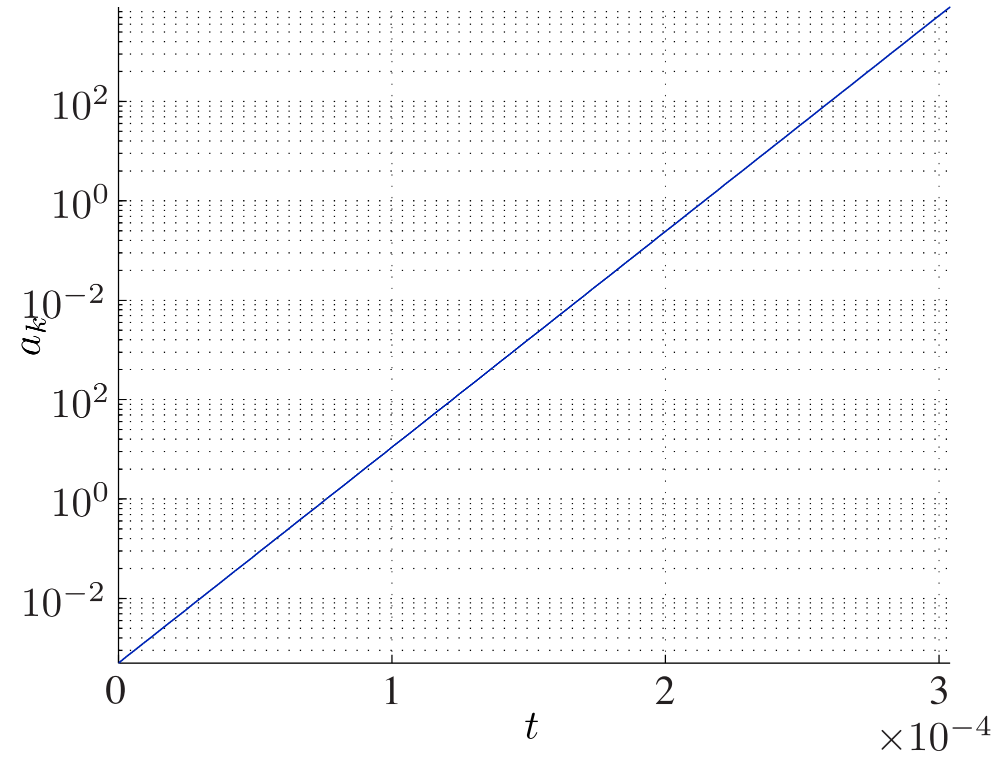
$$a_k(t) = \frac{1}{\sqrt{\exp(-2Ht) + (k/K)^2}}$$

$$a_k(t) \neq \exp(2\,H\,t) + (k/K)^2$$

Emergent scale factor during inflation

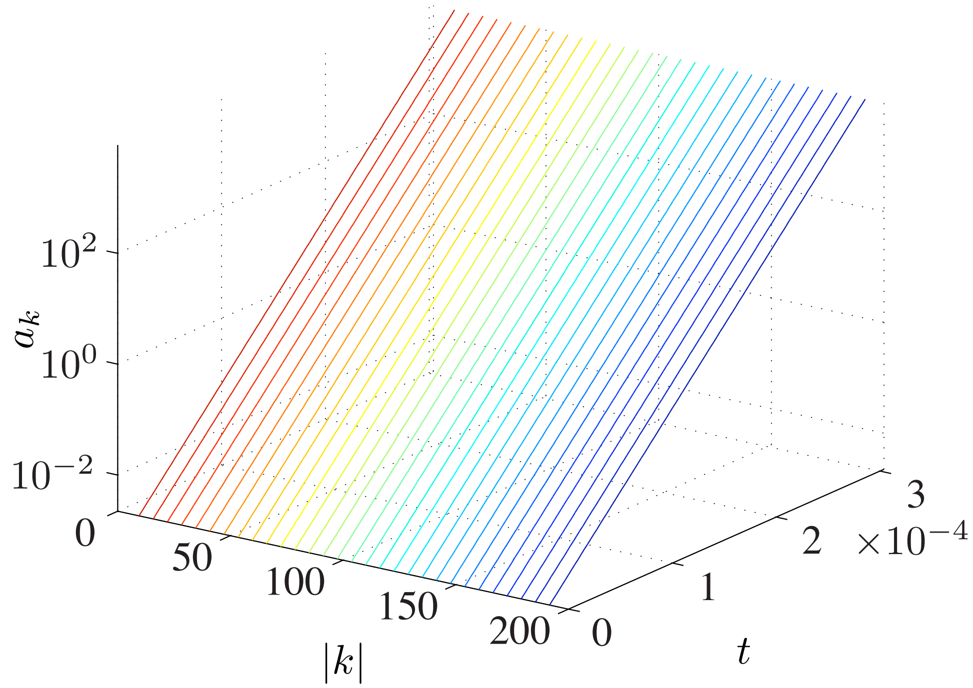


(a) Scale factor without quantum pressure effects ; $t_s = 1 \times 10^{-5}$.

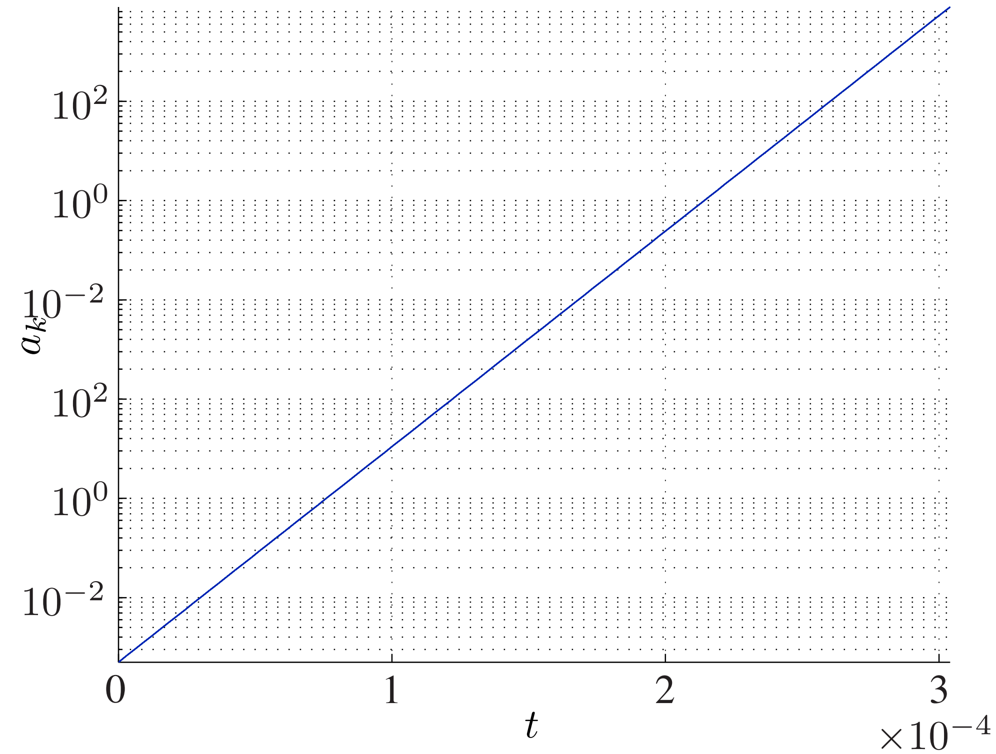


(b) Scale factor without quantum pressure effects; $t_s = 1 \times 10^{-5}$.

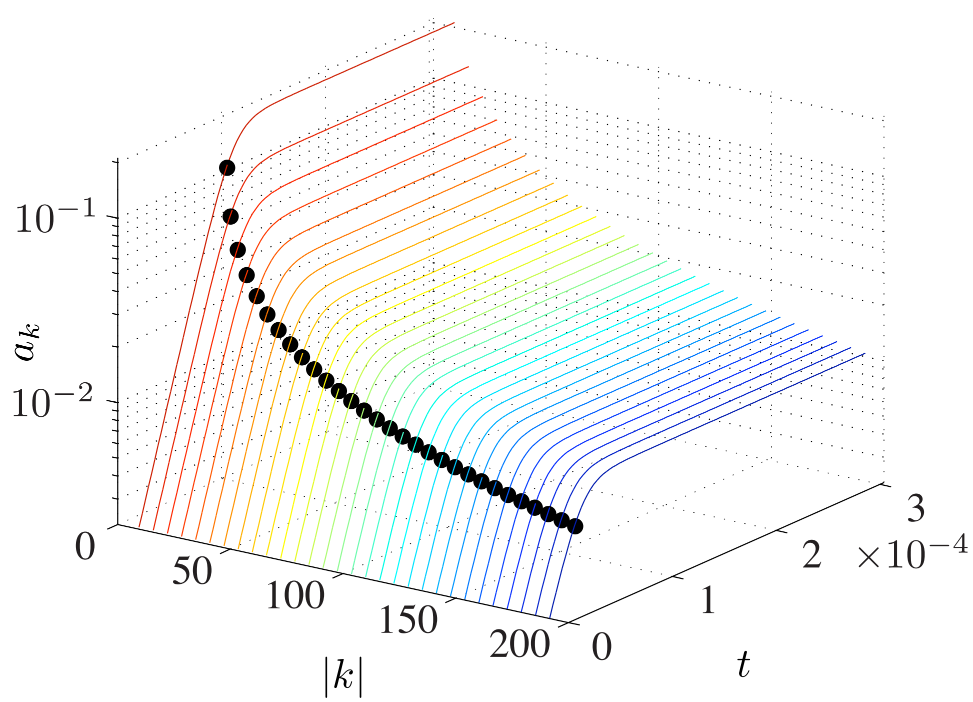
Emergent scale factor during inflation



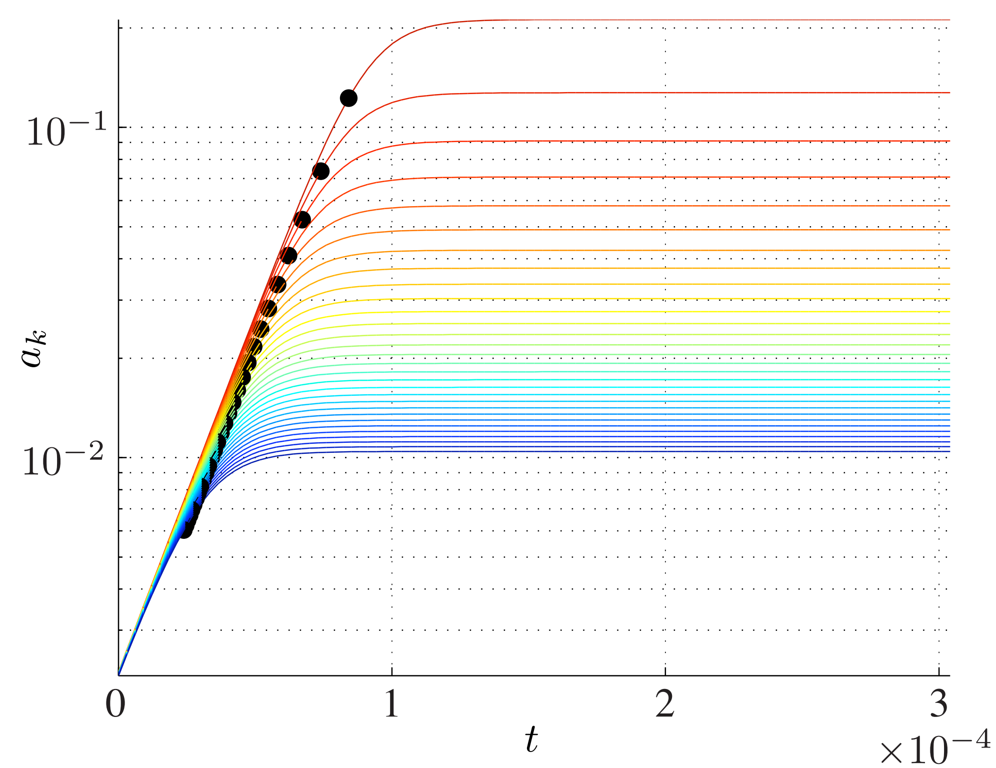
(a) Scale factor without quantum pressure effects ; $t_s = 1 \times 10^{-5}$.



(b) Scale factor without quantum pressure effects; $t_s = 1 \times 10^{-5}$.



(c) Scale factor quantum pressure effects ; $t_s = 1 \times 10^{-5}$.



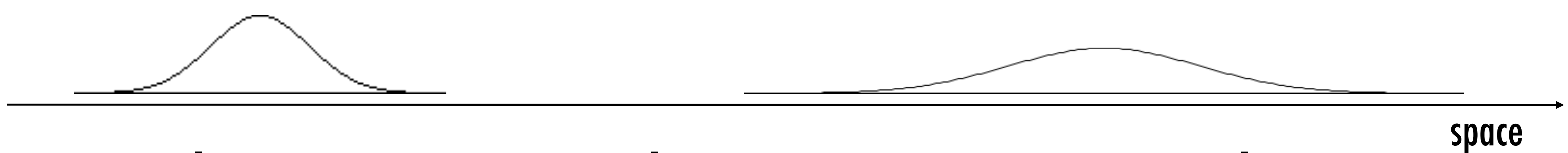
(d) Scale factor quantum pressure effects; $t_s = 1 \times 10^{-5}$.

Cosmological horizons

Phase velocity: $c_{\mathrm{phase}} := \frac{\omega_k}{k} = c_0\sqrt{\exp(-2Ht) + (k/K)^2}$

Beyond the hydrodynamic limit: phase velocity ≠ group velocity

Group velocity: $c_{\mathrm{group}} := \frac{\partial \omega_k}{\partial k} = \frac{c_0^2 \exp(-2Ht) + 2(k/K)^2}{\sqrt{c_0^2 \exp(-2Ht) + (k/K)^2}}$



Maximum distance a signal sent at t_0 can travel:

$$r_{\max} = \lim_{t \to +\infty} \int_{t_0}^{t} c_{\mathrm{group}}\, dt$$

→ within the hydrodynamic limit

$$\lim_{K \to \infty} r_{\max} = \lim_{t' \to +\infty} \int_{t_0}^{t'} \lim_{K \to \infty} c_{\mathrm{group}}\, dt = \frac{c_0}{H}$$

How far can a signal travel in a *real* BEC..?

$$r_{\max} = \lim_{t \to +\infty} \int_{0}^{t} c_{\mathrm{group}}\, dt. = \infty$$

No cosmological horizon..

Hubble parameter?

Hubble frequency?

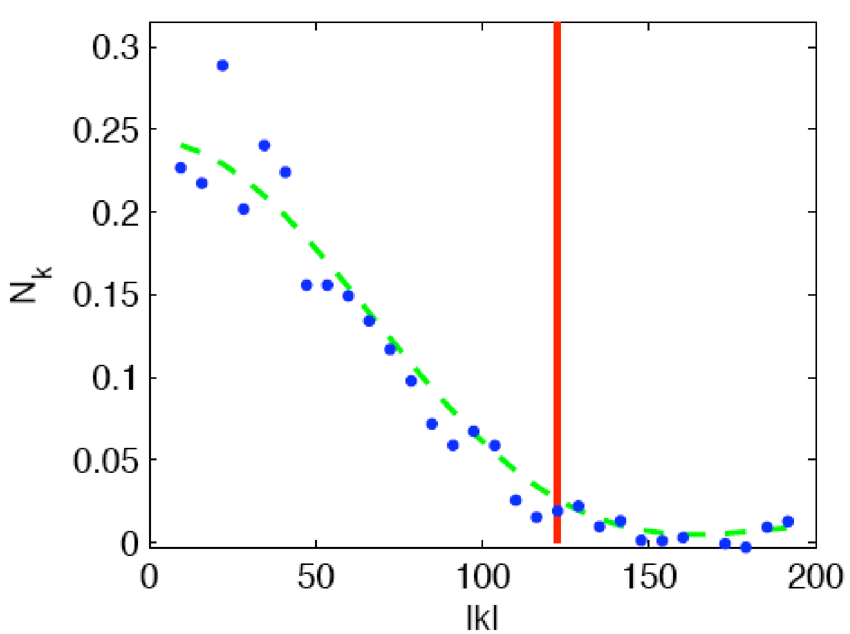
$$H := \frac{\dot{a}(t)}{a(t)} \quad \rightarrow \quad H_k = \frac{\dot{a}_k(t)}{a_k(t)}$$

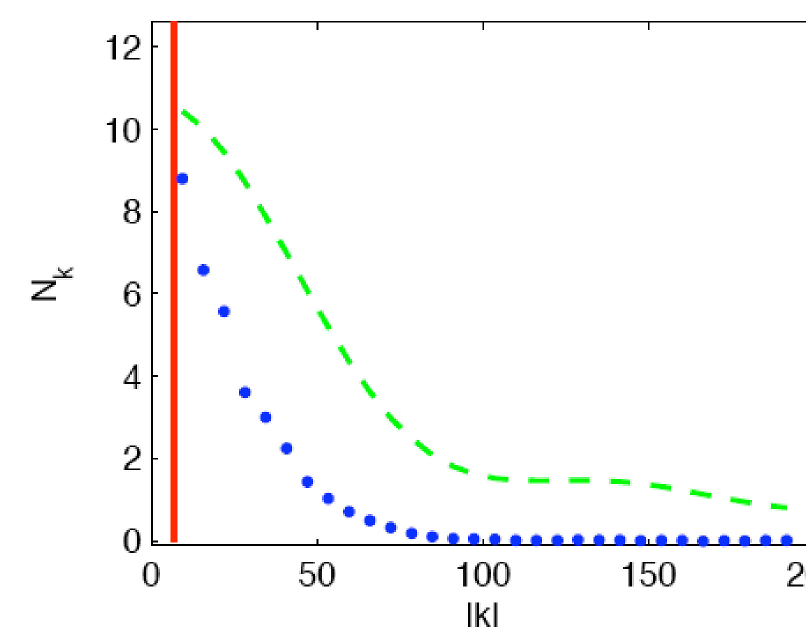
Modified **Hubble** parameter: $$H_k = H \, \frac{\exp(-2H\,t)}{\exp(-2H\,t) + (k/K)^2}$$

Early times… $$\lim_{t\rightarrow -\infty} H_k(t) \rightarrow H$$

Late times… $$\lim_{t\rightarrow +\infty} H_k(t) \rightarrow H\,K^2\,(\exp(-H\,t)/k)^2 \rightarrow 0$$

time





Frequency ratio

Equation of quantum modes in the hydrodynamic limit:

$$\ddot{\hat{\chi}}_k(t) + \underbrace{\left(\omega_k(t)^2 - H^2\right)}\hat{\chi}_k(t) = 0$$

Sign determines nature of quantum modes!

Characteristic value of particle production in the hydrodynamic limit: $R_k(t) = \frac{\omega_0}{H}\exp(-Ht)$

$R_k \gg 1$ sub Hubble "horizon" modes $\ddot{\hat{\chi}}_k(t) + \omega_k(t)^2\hat{\chi}_k(t) = 0$

$R_k = 1$ Hubble "horizon" crossing

$R_k \ll 1$ super Hubble "horizon" modes $\ddot{\hat{\chi}}_k(t) - H^2\hat{\chi}_k(t) = 0$

Modified frequency ratio

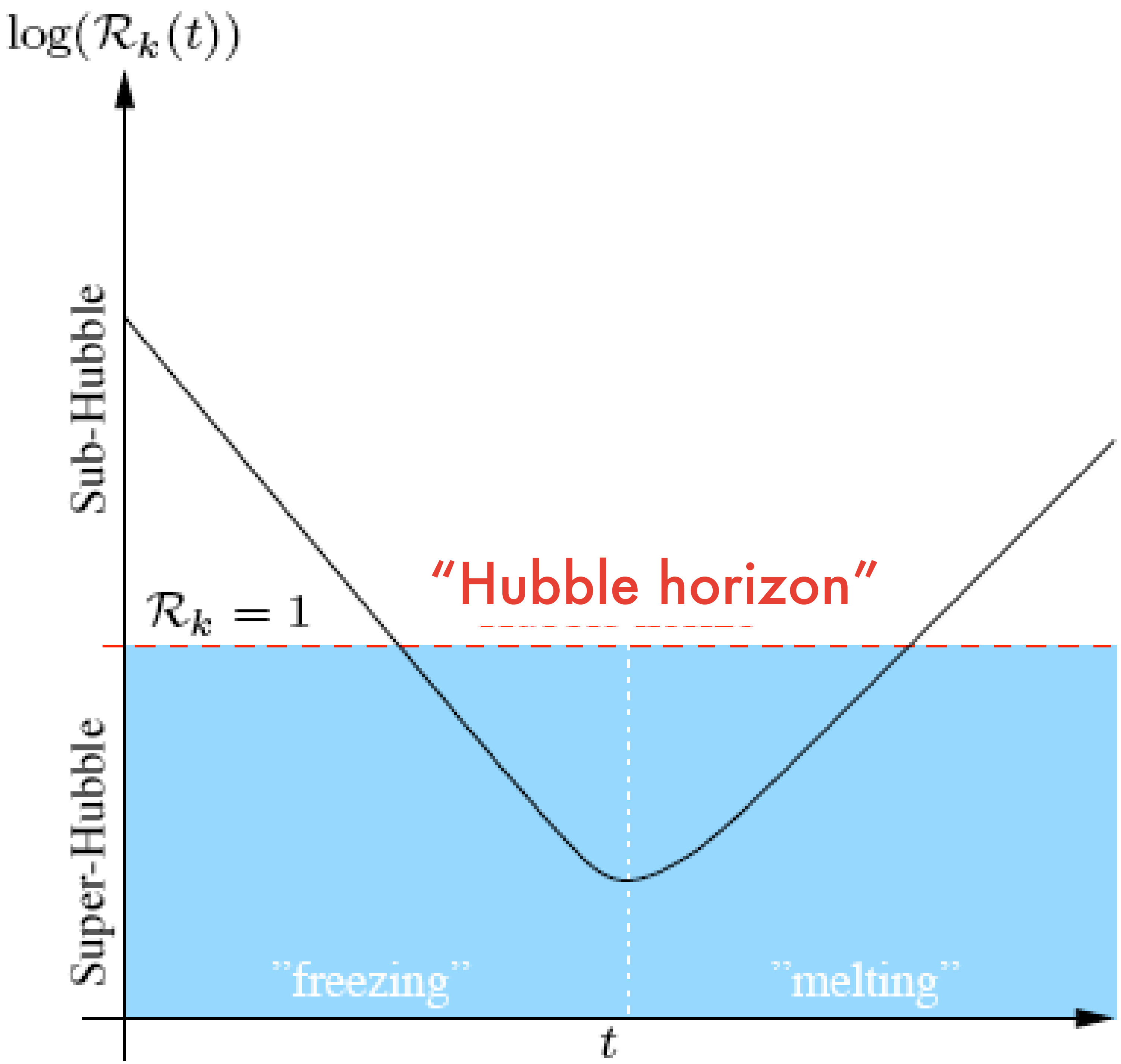
Modified frequency ratio:

$$\mathcal{R}_k(t) = \frac{\omega_k(t)}{H_k(t)} = \frac{\omega_0}{H} \frac{(\exp(-2H\,t) + (k/K)^2)^{3/2}}{\exp(-2H\,t)}\,.$$

Turning point: $t_{\rm turn} = \dfrac{\ln(K^2/(2\,k^2))}{2H}$

$$\mathcal{R}_k(t_{\rm turn}) = \frac{3\sqrt{3}}{2}\,\frac{\gamma_{\rm qp}}{H}\,k^2$$

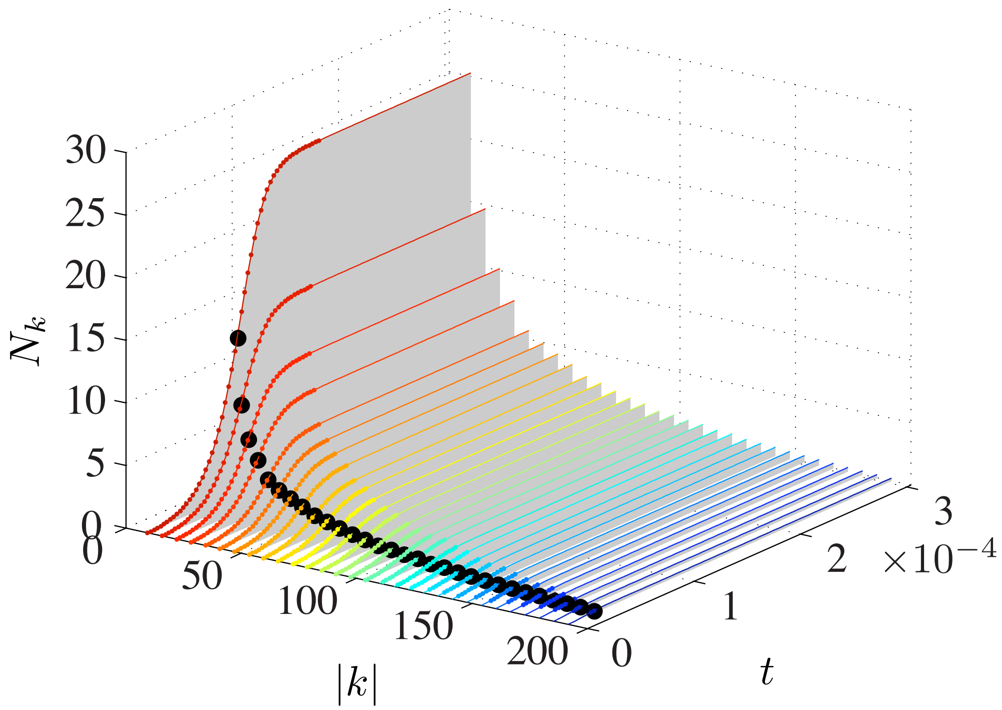
Freezing and melting of quantum modes



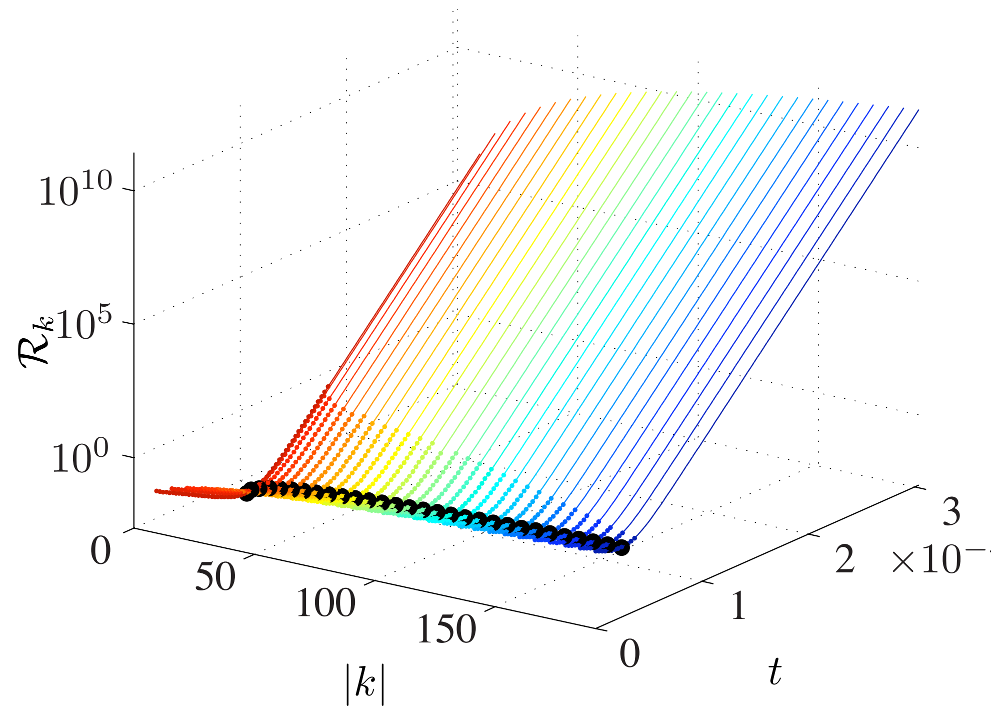
Crossing and re-entering of "Hubble horizon" during inflationary epoch! Inflation comes naturally to an end.

Can one always understand the particle production in terms of the frequency ratios?

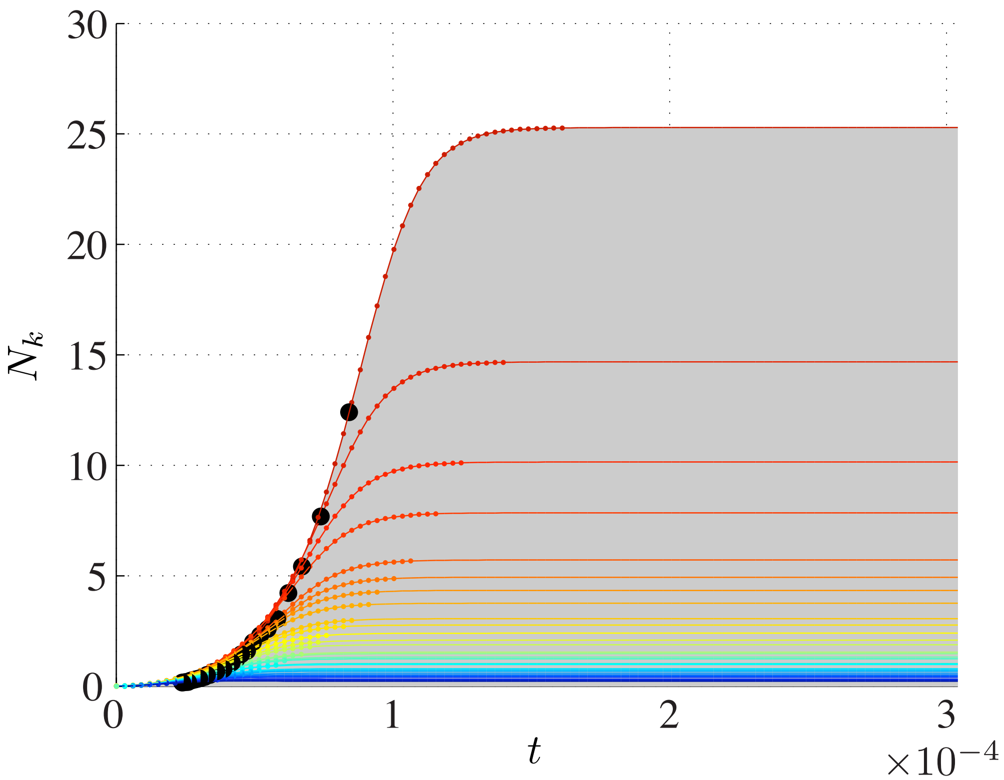
Simulations versus quantitative predictions



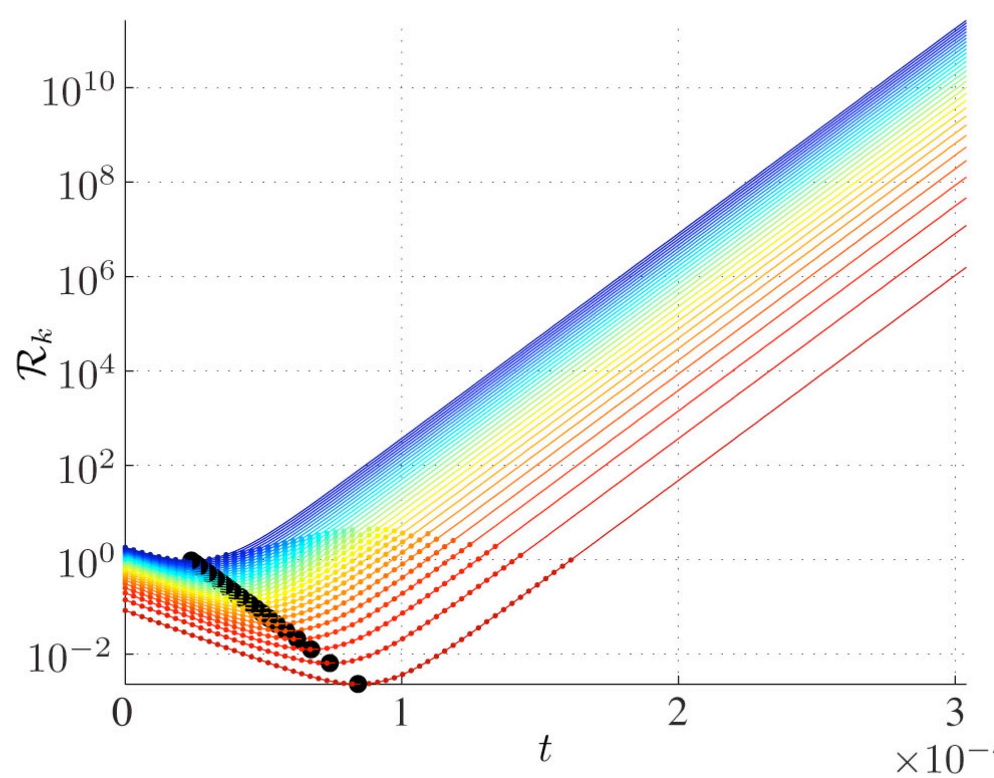
(a) $N_k(t)$.



(b) $\mathscr{R}_k(t)$.



(c) $N_k(t)$ projected onto the t-N_k plane.



(d) $\mathscr{R}_k(t)$ projected onto the t-$\mathscr{R}_k$ plane.

Planck-length during inflation

Lorentz symmetry breaking due to a preferred frame (in our specific spacetime) has non-negligible effects on the particle production process as the preferred frame is static!

In some sense we are dealing with a “relative scale-shift” between the two frames, a time-dependent “Planck-scale”,

$$\ell_{\rm Planck}(t) = 1/(K\sqrt{b(t)})$$

that enters the scale factor $a_k(t) = a_k(t)/\sqrt{1+k^2\ell_{\rm Planck}(t)^2}$ of the universe and the “effective” Hubble parameter $H_k(t) = H/(1+k^2\ell_{\rm Planck}(t)^2)$ in an unexpected way. (The non-perturbative corrections have to be included at the level of the hydrodynamic fluid equations.

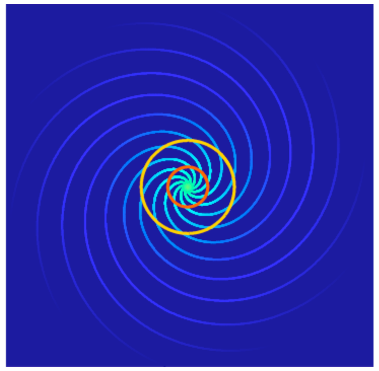
However it is possible to mimic an expanding universe and keep the effective Planck length constant:

$$a_{\rm FRW}(t) = a_{\rm FRW,0}\left(\frac{b_n(\tau)}{b_a(\tau)}\right)^{\frac{1}{2(d-1)}} \qquad \ell_{\rm Planck}(t) = \frac{\gamma_{\rm qp}}{c(t)} = \frac{\gamma_{\rm qp}}{c_0\sqrt{b_n(t)b_a(t)}} = \frac{\ell_{\rm Planck,0}}{\sqrt{b_n(t)b_a(t)}}$$

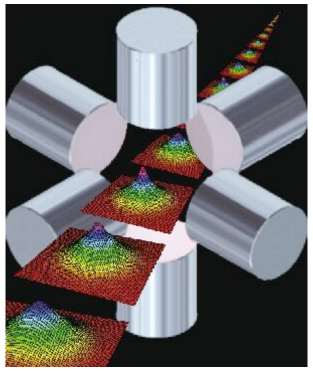
Singularity

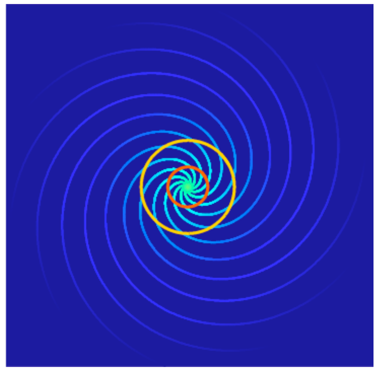

Singularity


Singularity




Singularity



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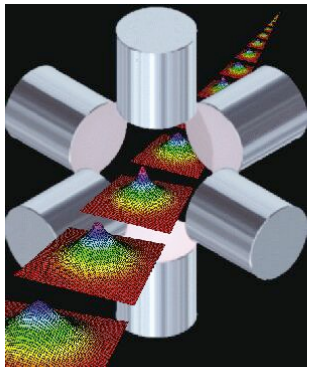
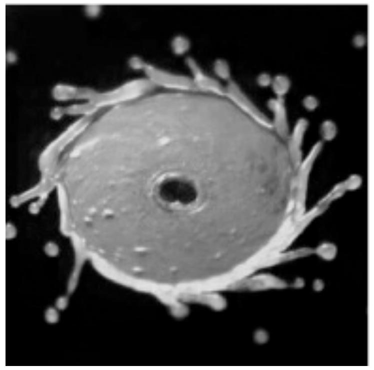
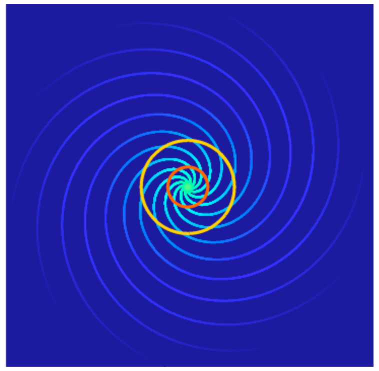
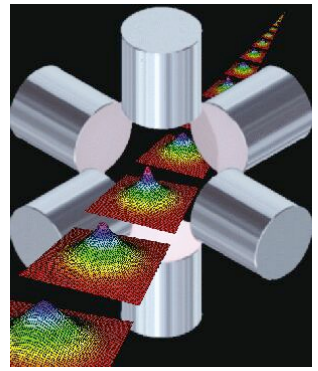
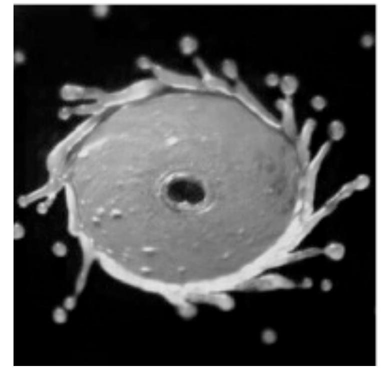
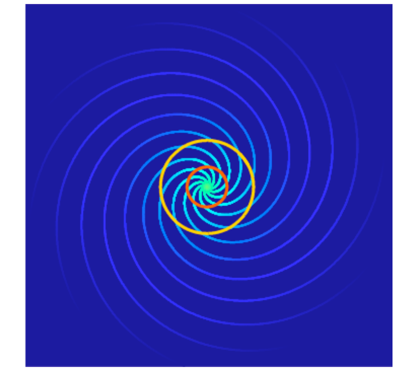










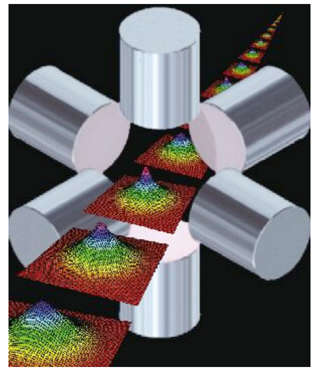


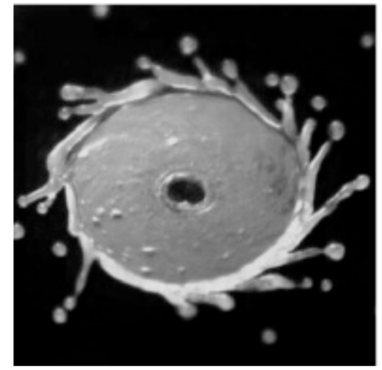
“Black Hole” Nucleation in a Splash of Milk

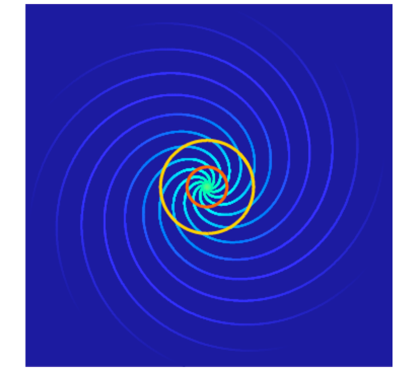
Laurent Courbin[1], James C. Bird[1], Andrew Belmonte[2]
& Howard A. Stone[1]

1. School of Engineering and Applied Sciences, Harvard University, USA
2. W. G. Pritchards Labs, Dept of Mathematics, Penn State University, USA

APS DFD, Gallery of Fluid Motion, 2007







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From emergent spacetimes to emergent gravity...?

Emergent gravity

Einstein dynamics:

$$G_{ab} = 8\pi G_{\mathrm{N}}\, T_{ab}$$

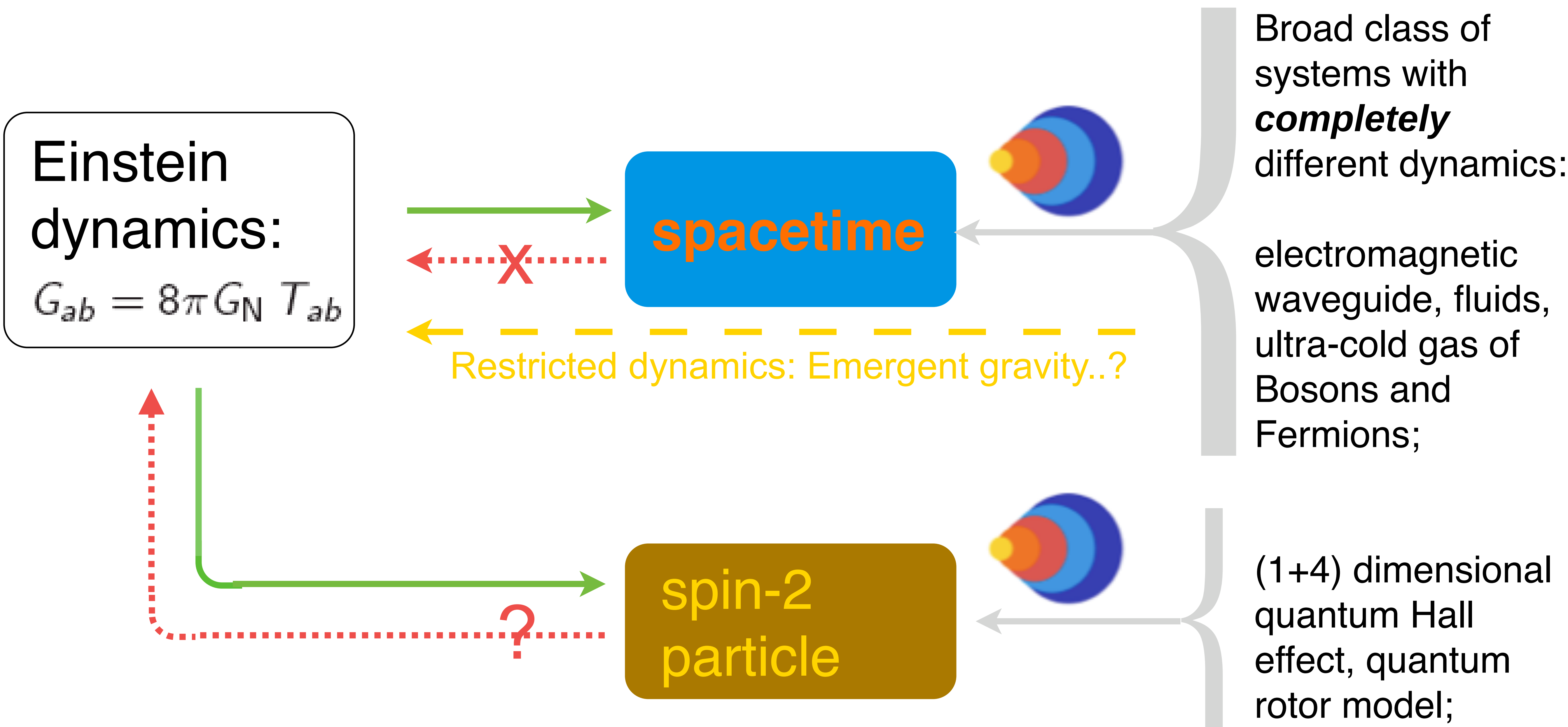
spacetime

Restricted dynamics: Emergent gravity..?

Broad class of systems with ***completely*** different dynamics:

electromagnetic waveguide, fluids, ultra-cold gas of Bosons and Fermions;

Emergent gravity



books on the **Emergent Spacetime / Analogue Models** for Gravity:

Artificial black holes (Novello, Visser, Volovik)

The universe in a helium droplet (Volovik)

Quantum Analogues: From phase transitions to black holes and cosmology (Unruh, Schuetzhold)

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